Spectrum of charmed mesons from dynamical anisotropic lattices

M. B. Oktay
Trinlat Collaboration
Trinity College, Dublin

Troia07, Çanakkale
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Motivation

▷ Charmonium $c\bar{c}$ well established below the $D\bar{D}$ threshold.

▷ New charmonium states have been observed in the recent years. X(3872) observed by Belle,CDF $J^{PC} = 1^{++}$ or 2^{-+}, molecule ? X(3940) [$\eta^{''}(3S)$?], Y(3940) [$\chi_{c_1}(2P)$?], Z(3930) [$\chi_{c_2}(2P)$?]

▷ Heavy Hybrid candidates Y(4360) and other states such as $1^3D_3, \ldots, 1^3F_2$ etc.

▷ Experimental efford is underway to detemine the D$_s$ states, e.g., D$_{sJ}$ [Babar hep-ex/0304021, Cleo hep-ex/0305017]

▷ Spectrum studies provide us with an important test of lattice methods that are very similar to those used for CKM determinations in other systems e.g., D-meson system, B$_s$.

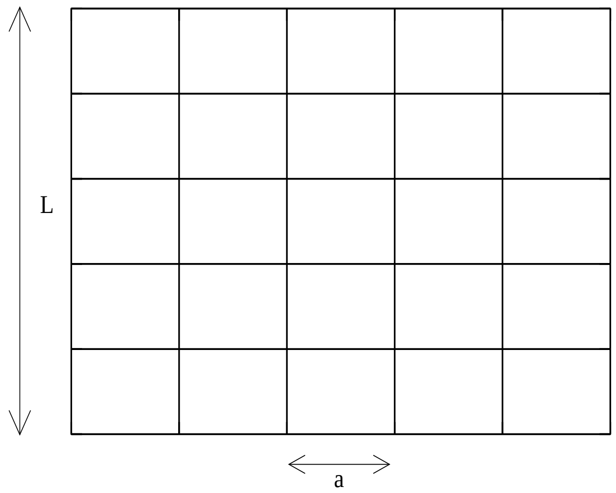
▷ We would like to calculate the charmed-meson spectrum with high precision with practical "all-to-all" method together with "anisotropic" lattices.

Lattice

$$Z = \int DA_\mu D\psi D\bar\psi \, \text{exp}[-S_g - S_f] = \int DA_\mu (\text{det}\, M_f) \, \text{exp}[-S_g]$$

and the observables

$$\langle \mathcal{O} \rangle = \int DA_\mu D\psi D\bar\psi \, [\mathcal{O}] e^{-S} / \int DA_\mu D\psi D\bar\psi e^{-S}$$



- ψ, $\bar\psi$: Sites
- $U_\mu(x) = \text{exp}[igA_\mu]$: Links
- Gauge Invariance

Anisotropic Lattices

$a_s = \xi\ a_t$ with $\xi > 1$ where a_s spatial and a_t temporal lattice spacing. The spatial lattice extent should be large enough to accommodate the heavy-light state.

□ Better signal : Correlators for the heavy-light sytem become noisy at large t. This make it difficult to resolve a plateau in the effective mass plots. Increase in the number of time slices allow to resolve the signal early and reduces the statistical error.

□ Smaller discretization errors : large energy scale of order m_Q appears in the temporal component so one expects errors $\mathcal{O}[(a_t m_Q)^n]$.

□ Difficult to determine the lattice spacing: 2 lattice spacings need to be determined ! Morningstar *et. al.* Phys. Rev. D60 034509 (1999).

Fermion Action (Fine Wilson Coarse Hamber-Wu)

$$
\begin{aligned}
M\ \psi(x) = \frac{1}{a_t}\Bigg\{ & \left(\mu_r m a_t + \frac{9}{4\xi_q^{(0)}} + 1 + \frac{a_t^2 g}{4}\sigma_{i0}E_i\right)\psi(x) \\
& -\frac{1}{2u_t}\left[(1-\gamma_0)U_t(x)\psi(x+\hat{t}) + (1+\gamma_0)U_t^\dagger(x-\hat{t})\psi(x-\hat{t})\right] \\
& -\frac{1}{\xi_q^{(0)}}\sum_i \Bigg[\frac{1}{u_s}\left(\frac{1}{2}-\frac{2}{3}\mu_r\gamma_i\right)U_i(x)\psi(x+\hat{\imath}) \\
& +\frac{1}{u_s}\left(\frac{1}{2}+\frac{2}{3}\mu_r\gamma_i\right)U_i^\dagger(x-\hat{\imath})\psi(x-\hat{\imath}) \\
& -\frac{1}{u_s^2}\left(\frac{1}{8}-\frac{1}{12}\mu_r\gamma_i\right)U_i(x)U_i(x+\hat{\imath})\psi(x+2\hat{\imath}) \\
& -\frac{1}{u_s^2}\left(\frac{1}{8}+\frac{1}{12}\mu_r\gamma_i\right)U_i^\dagger(x-\hat{\imath})U_i^\dagger(x-2\hat{\imath})\psi(x-2\hat{\imath})\Bigg]\Bigg\}
\end{aligned}
$$

where $\mu_r = 1 + \frac{1}{2}ma_t$. J. Foley *et. al.*, hep-lat/0405030.

Gauge Action (Two-Plaquette Symanzik Improved)

$$S_g = \frac{\beta}{\xi_g}\left[\frac{5(1+w)}{3u_s^4}\Omega_s - \frac{5w}{3u_s^8}\Omega_s^{(2t)} - \frac{1}{12u_s^6}\Omega_s^{(R)}\right] + \beta\xi_0\left[\frac{4}{3u_s^2u_t^2}\Omega_t - \frac{1}{12u_s^4u_t^2}\Omega_t^{(R)}\right]$$

where

$$\Omega_s = \sum_{x,i>j}\left[1-P_{ij}\right], P_{ij} = \text{Spatial Plaquette}$$

$$\Omega_t = 1 - \text{time-like plaquette}$$

$$\Omega_s^{(2t)} = \frac{1}{2}\sum_{x,i>j}\left[1 - P_{ij}(x)P_{ij}(x+\hat{t})\right]$$

$$\Omega_{s/t}^{(R)} = 2\times 1\text{rectange in (i,j/t) plane} \quad (1)$$

Morningstar and Peardon, NPBP **83–84**, 887 (2000).

All-to-all Propagators

Point Propagators : restrict the physics, interpolating operator basis and lose information stored in the gauge configurations.

All-to-all Propagators :Expensive.

Hybrid Method : Define the Hermitian Dirac Operator $Q = \gamma_5 M$ where M is the Dirac operator.

$$Q^{-1} = \sum_{i}^{N} \frac{1}{\lambda_{(i)}} v^{(i)} \otimes v^{(i)\dagger} \quad \text{where} \quad M v^{(i)} = \lambda_{(i)} v^{(i)}$$

$$= \underbrace{\sum_{i=1}^{N_{ev}} \frac{1}{\lambda_i} v^{(i)} \otimes v^{(i)\dagger}}_{Q_0^{-1}} + \underbrace{\sum_{i=N_{ev}+1}^{N} \frac{1}{\lambda_i} v^{(i)} \otimes v^{(i)\dagger}}_{Q_1^{-1}} \tag{2}$$

and correct the truncation by a stochastic method : Dilution.

Estimate Q_1^{-1} using the stochastic method $Q_1^{-1} = \langle\langle \psi \otimes \eta^\dagger \rangle\rangle$ with N_r noise vectors $\left\{\eta_{[1]}, \dots, \eta_{[N_r]}\right\}$. The solutions : $\psi_{[r]} = Q_1^{-1}\eta_{[r]}$.

$$\eta \;=\; \sum_j \eta^{(j)} \qquad j \text{ : dilution index}$$

where we can dilute in time, space, color and spin resulting in the exact all-to-all propagator in a finite number of steps. We obtain a hybrid list for the source and solution vectors :

$$w^{(i)} \;=\; \left\{\frac{v^{(1)}}{\lambda_1}, \dots, \frac{v^{(N_{ev})}}{\lambda_{N_{ev}}}, \eta^{(1)}, \dots, \eta^{(N)}\right\}$$
$$u^{(i)} \;=\; \left\{v^{(1)}, \dots, v^{N_{ev}}, \psi^{(1)}, \dots, \psi^{(N)}\right\}$$

and

$$M^{-1} = \sum_i u^{(i)}(x, x_0) \otimes w^{(i)}(y, y_0)\gamma_5$$

[Foley, *et. al.* Comput. Phys. Commun.172:145-162,2005.]

Parameters

Gauge action	β	1.508
(TSI 3+1)	ξ_g	8.42
	ω	3.0
	u_s^4	0.32
Fermion action	m_{sea}	-0.057
(fWcHW)	ξ_q	7.43
	u_s, u_t	1
	Stout-links	2x0.22 spatial
Charm quarks	m_c	0.117
	ξ_q	5.90
Physical values	a_s	$\sim$ 0.17 fm
	ξ_r	6.00
	m_π/m_ρ	$\sim$ 0.54
	Lattice Size	$12^3 \times 80$
	Dilution	time+space(even-odd)

R. Morrin *et. al.*,Phys.Rev.D74:014505,2006 [hep-lat/0604021]

Operators

All-to-all propagators allow us to easily construct complicated operators :

$$O_{\Gamma}^{(i,j)}(x,t) = w_{[1]}^{(i)}(x,t)^{\dagger}\Gamma(\Pi U)u_{[2]}^{(j)}(x,t)$$

from which we can construct the correlators

$$C(t_x,t_y) = \sum_{i,j} O_{[1,2]}^{(i,j)}(t_x) O_{[2,1]}^{(j,i)}(t_y)$$

Example :

$^3S_1(1^-)$ $\quad D_s^{\star}$ $\quad \gamma_1 \sum_i [U_i(x)u(x+i) + Ux^{\dagger}(x-i)u(x-i)]$

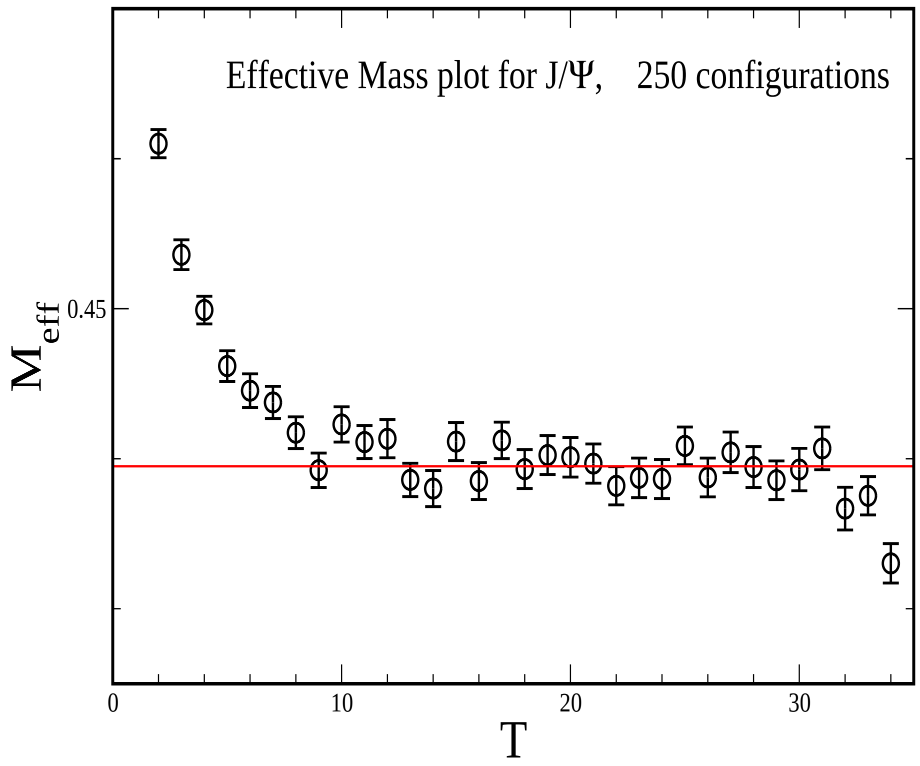
Notation:

$$p_j\psi(x) = U_j(x)\psi(x+j) - U_j^{\dagger}(x-j)\psi(x-j)$$
$$s_j\psi(x) = U_j(x)\psi(x+j) + U_j^{\dagger}(x-j)\psi(x-j)$$

J^{PC}	$^{2S+1}L_J$	STATE	OPERATORS
0^{-+}	$^1S_0$	η_c,η_c'	$\gamma_5,\gamma_5 \sum_i s_i$
1^{--}	$^3S_1$	$J/\psi,\psi(2S)$	$\gamma_j,\gamma_j \sum_i s_i$
1^{+-}	$^1P_1$	h_c,h_c'	$\gamma_i\gamma_j,\ \gamma_5 p_j$
0^{++}	$^3P_0$	χ_{c0},χ_{c0}'	$1,\vec{\gamma}\cdot\vec{p}$
1^{++}	$^3P_1$	χ_{c1},χ_{c1}'	$\gamma_5\gamma_i,\vec{\gamma}\times\vec{p}$
2^{++}	$^3P_2$	χ_{c2},χ_{c2}'	$\vec{\gamma}\times\vec{p},\ \gamma_1 p_1-\gamma_2 p_2$ $2\gamma_3 p_3-\gamma_1 p_1-\gamma_2 p_2$
2^{-+}	$^1D_2$	1^2D_2	$\gamma_5(s_1-s_2),\gamma_5(2s_3-s_1-s_2)$
2^{--}	$^3D_2$	1^3D_2	$\gamma_j(s_i-s_k),\gamma_1 t_1-\gamma_2 t_2,$ $2\gamma_3 t_3-\gamma_1 t_1-\gamma_2 t_2$
3^{--}	$^3D_3$	1^3D_3	$\vec{\gamma}\cdot\vec{t}$
1^{-+}	Hybrid	$q\bar{q}g$	$\vec{\gamma}\times\vec{u}$

[P. Lacock, C. Michael, P. Boyle and P. Rowland, Phys. Rev. D54, 6997 (1996)]

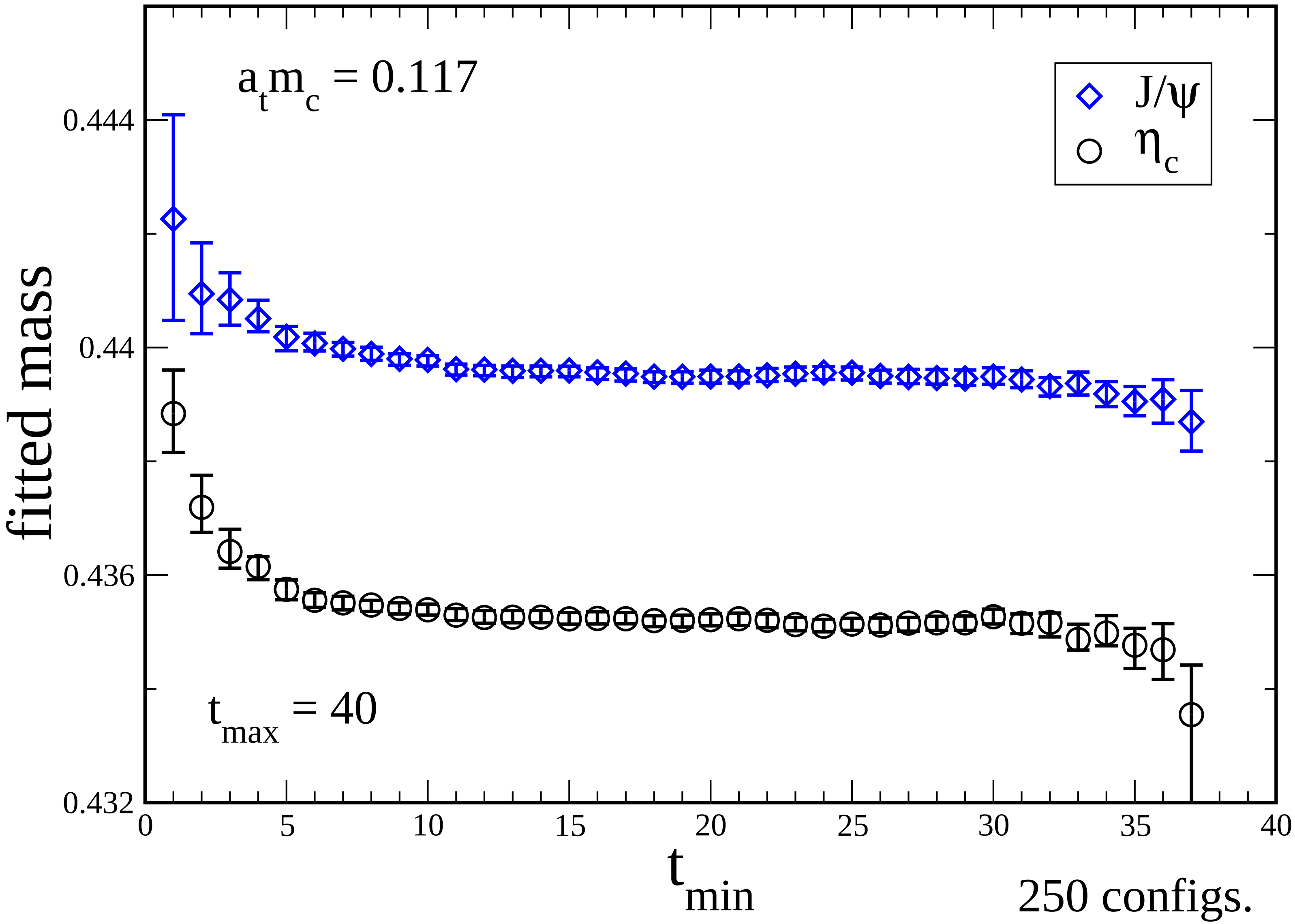
All-to-all method introduces local fluctuations which results in large errors in the effective mass plots and difficult to identify the plateau region.



However, this does not affect the fits. In order to resolve this problem, we do "sliding window" plots. Conditions : χ^2/n.d.f<2 and $Q>0.2$.

TrinLat @ TCD.

Sliding window analysis for η_c and J/ψ



Mass Difference

We can calculate the temporal lattice spacing from $\Delta M(^1P_1 - \bar{S})$:

$$\Delta M(\bar{S}) = \frac{1}{4}[M(^1S_0) + 3M(^3S_1)]$$

J^{PC}	Fitted Mass ($a_t E_0$)	(t_{min},t_{max})	χ^2/n.d.f	Q
0^{-+}	0.4352 ± 0.0001	(18,40)	1.10	0.31
1^{--}	0.4397 ± 0.0001	(18,40)	0.84	0.66
1^{+-}	0.5039 ± 0.0003	(17,29)	0.69	0.77

$\Delta M(^1P_1 - \bar{S}) \simeq 458 MeV$

$$a_t^{-1} = 7.22 \pm 0.03 \text{ GeV}$$

$$a_t = 0.0284 \pm 0.0002 \text{ fm}$$

Using $\xi_r \sim 6.0$:

$$a_s \simeq 0.17 \text{ fm}$$

Variational Analysis

In order to study the radial excitations, we adopt the "variational analysis " [C. Michael, Nucl. Phys. B 259, 58 (1985); Luscher and Wolf, NPB339, 222, (1990)]. The idea is to work with different interpolating operators $\mathcal{O}_\alpha$, $\alpha = 1, 2, \ldots, n$ and construct the matrix

$$C_{\alpha\beta} = \langle 0|\mathcal{O}_\alpha(t)\mathcal{O}^\dagger_\beta(0)|0\rangle$$

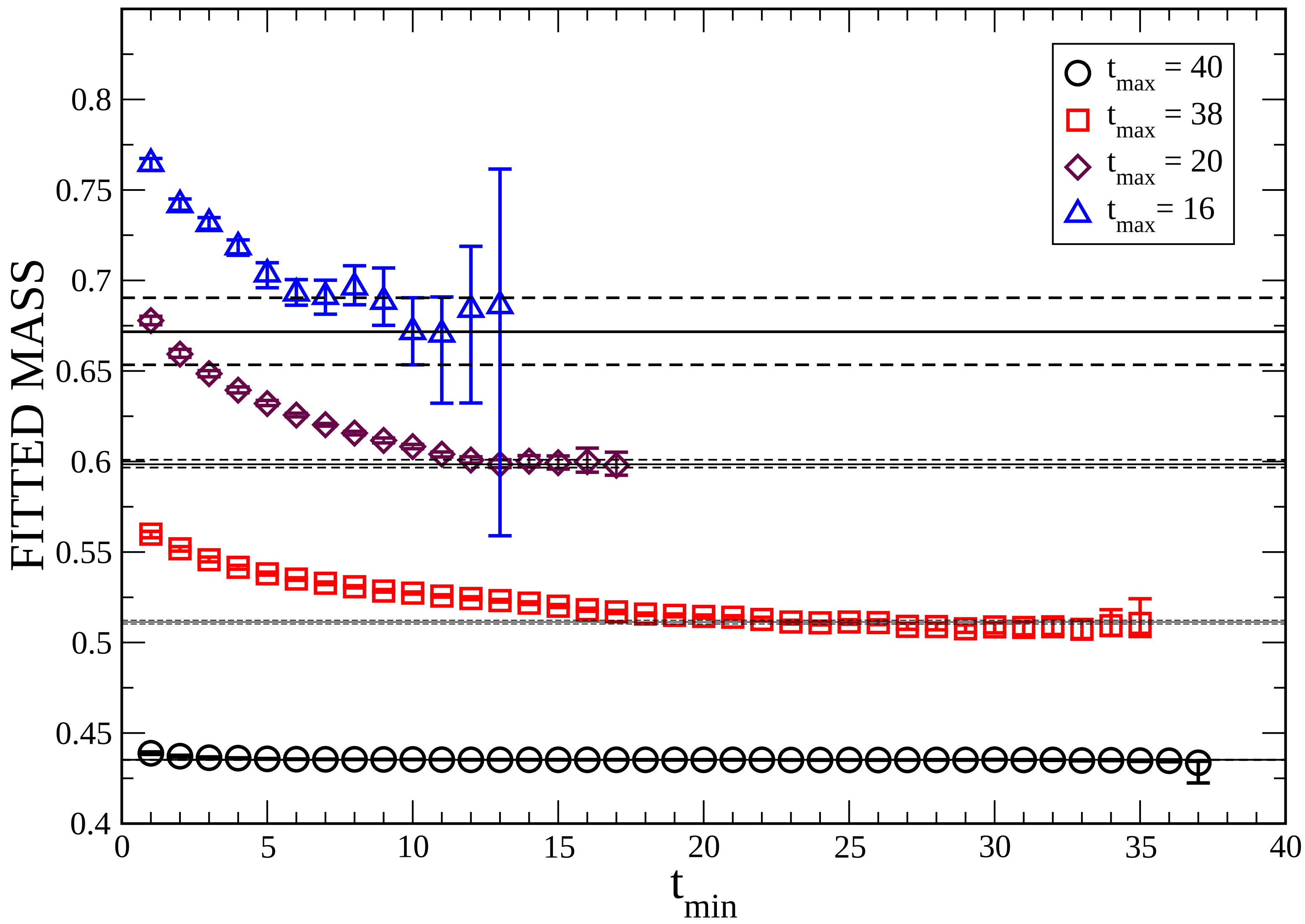
and λ_α are the eigenvalues of the matrix $C(t_0)^{-1/2}C(t)C(t_0)^{-1/2}$ where t_0 is some small reference time, $(\lambda_1 \geq \lambda_2 \geq \lambda_3 \ldots)$ and

$$\lim_{t\to\infty} \lambda_\alpha(t, t_0) = e^{-(t-t_0)E_\alpha}[1 + O(e^{(-t\Delta E_\alpha)}].$$

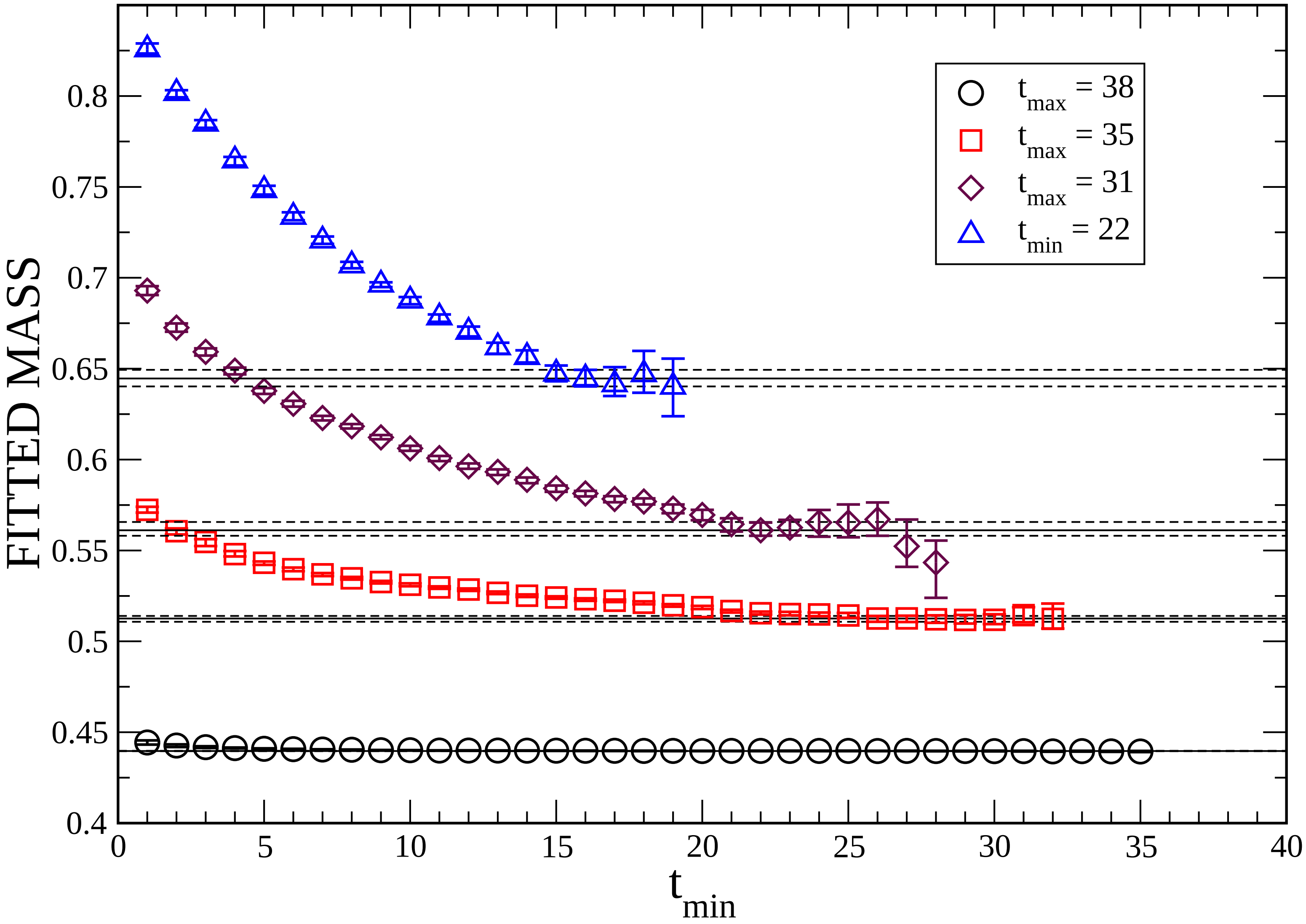
We perform single-state fits to the diagonal elements.

TrinLat @ TCD.

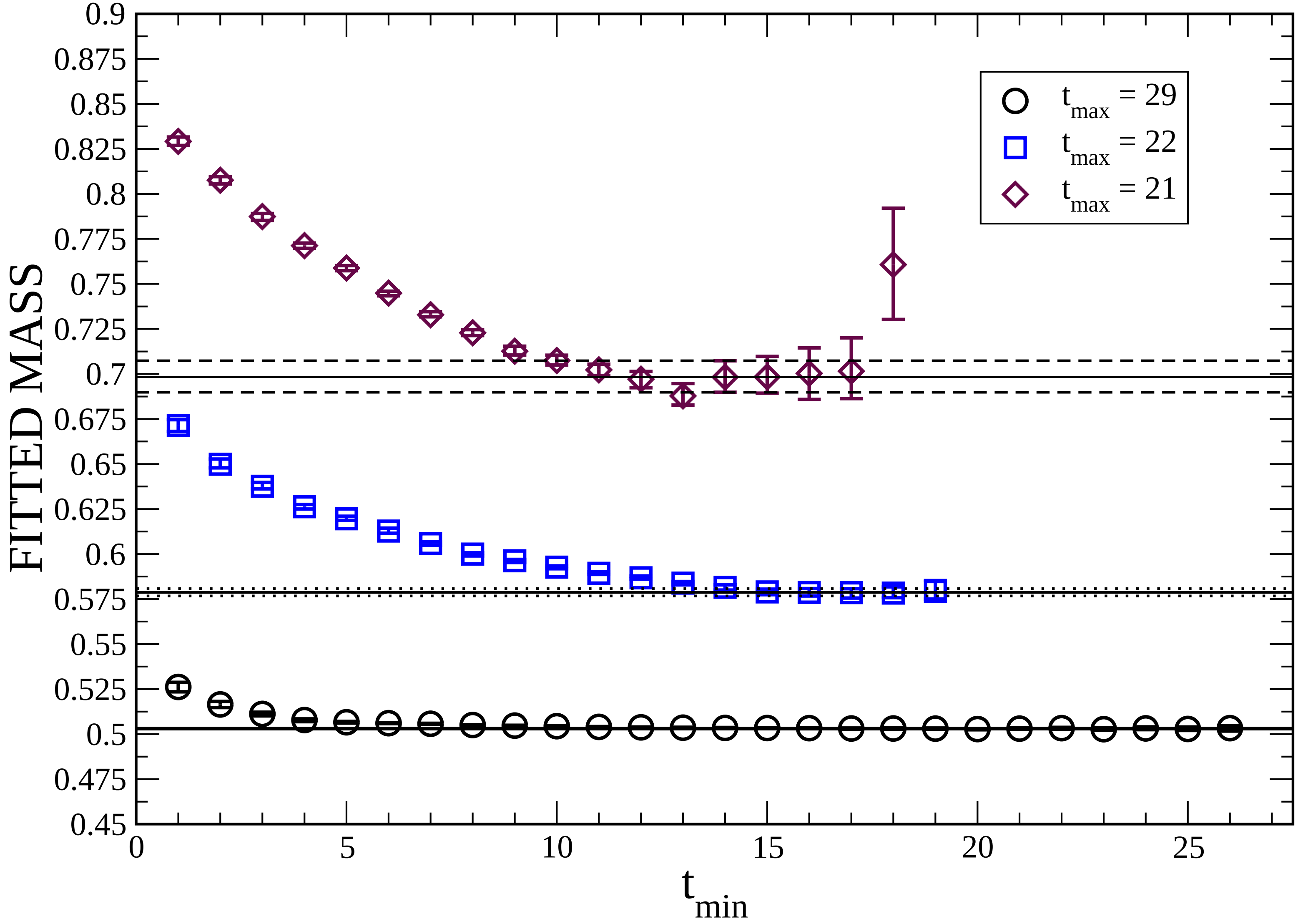
Sliding window plots for η_c, $\eta^{|}_c$ and $\eta^{||}_c$ (0^{-+})



Sliding window plots for J/ψ, $\psi^{|}$, $\psi^{||}$ and $\psi^{|||}$ (1^{--})

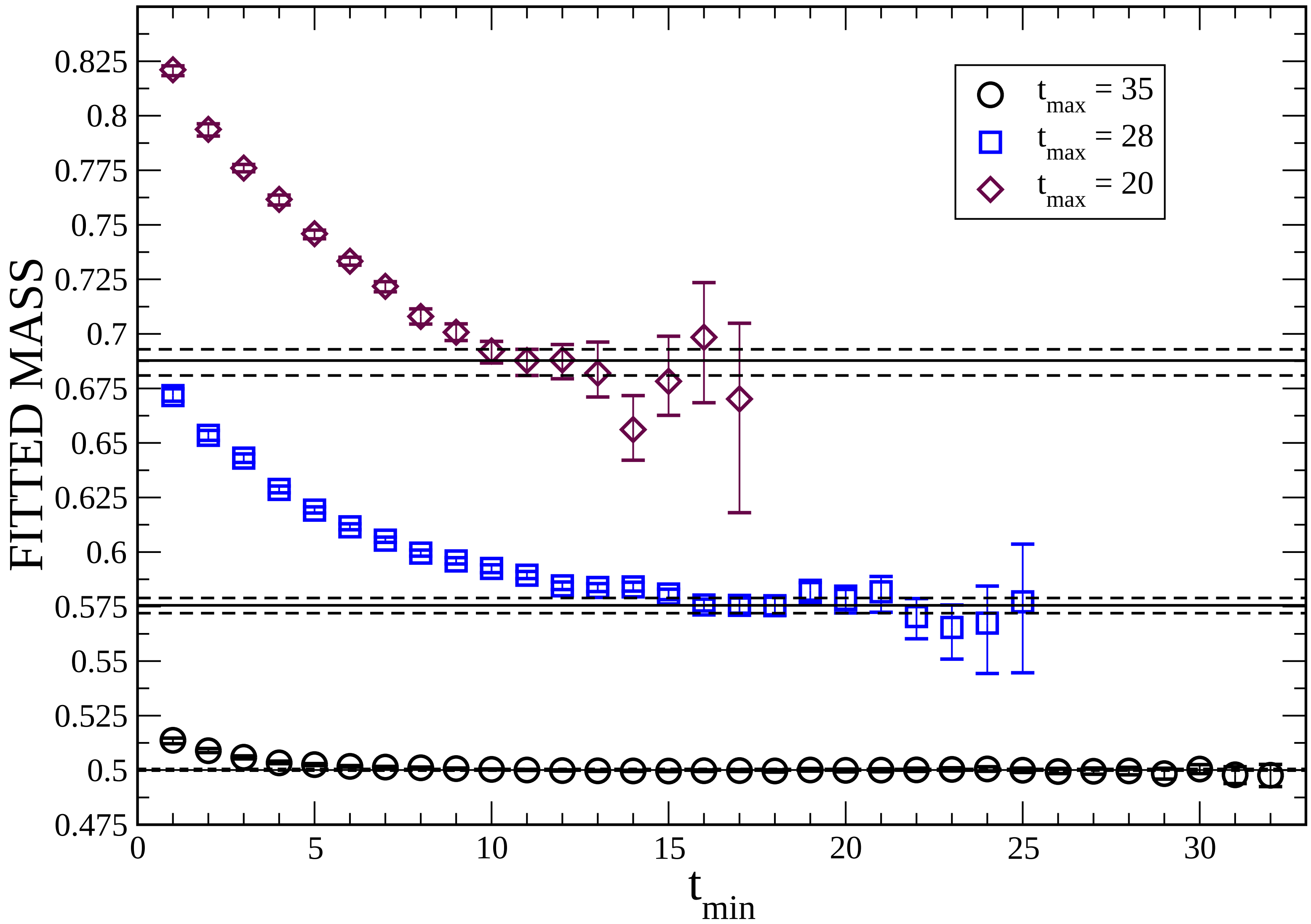


Sliding window plots for h_c, $h^{|}_c$ and $h^{||}_c$ (1^{+-})

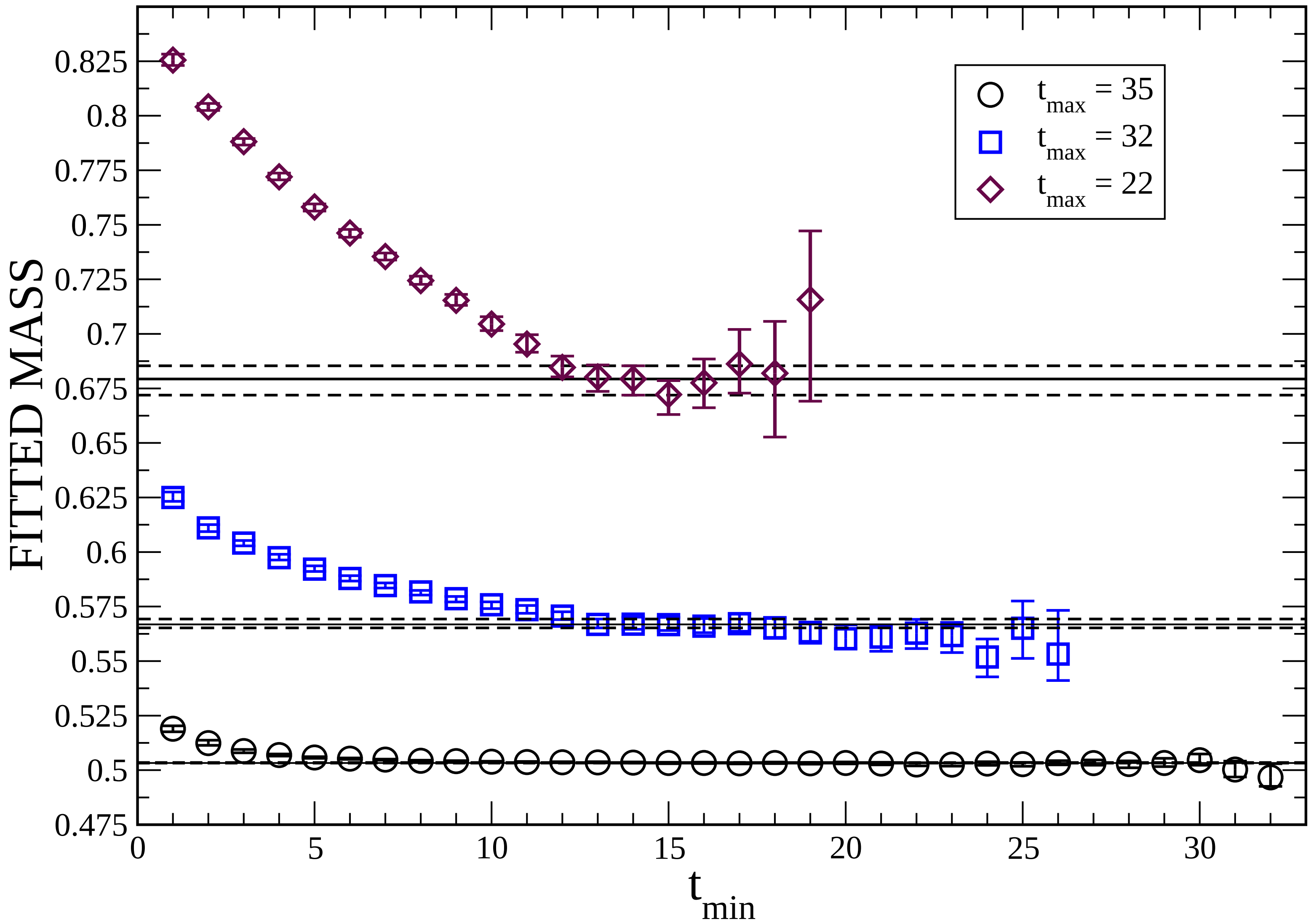


TrinLat @ TCD.

Sliding window plots for χ_{c_0}, $\chi^{|}_{c_0}$ and $\chi^{||}_{c_0}$ (0^{++})

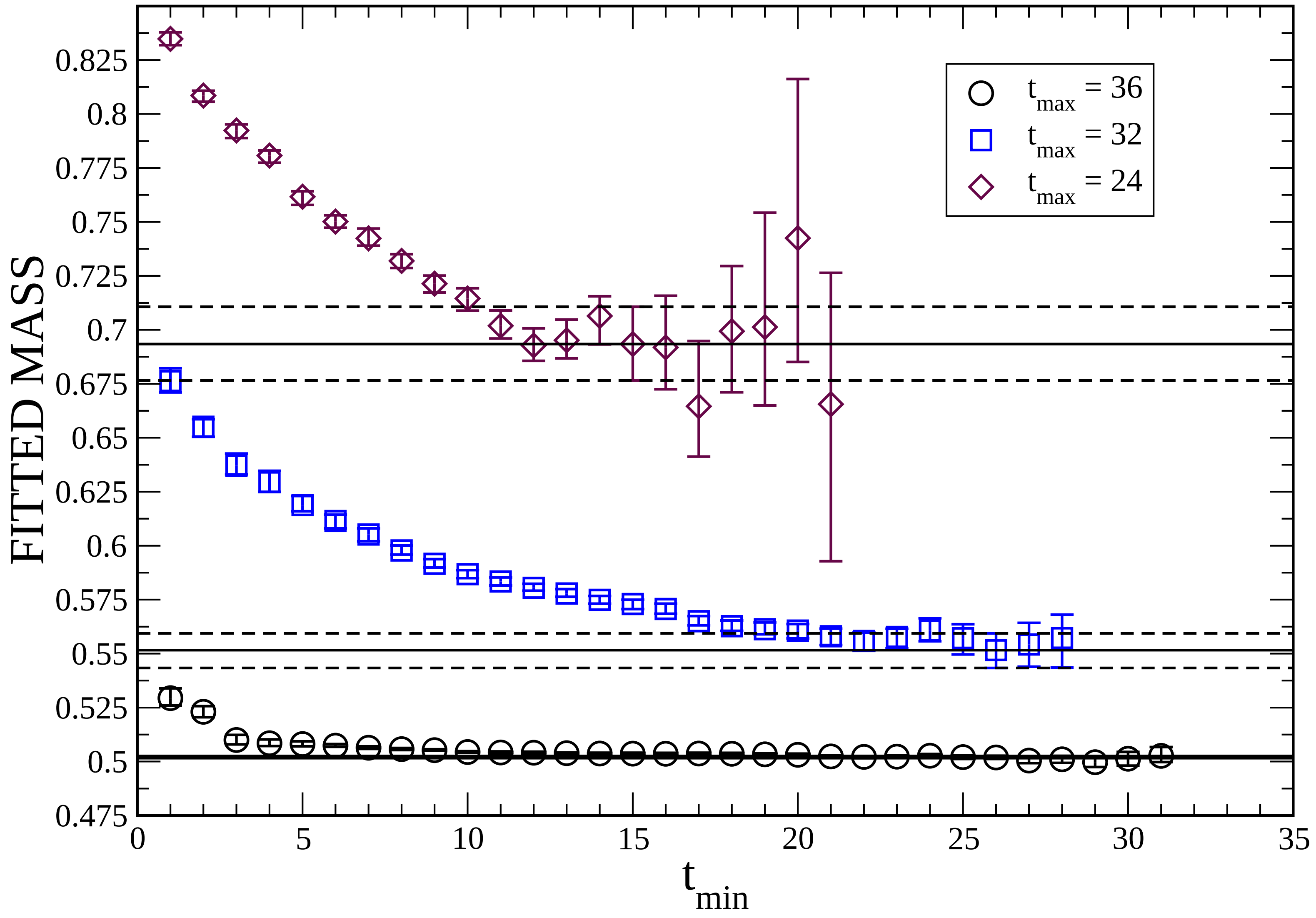


Sliding window plots for χ_{c_1}, $\chi^{|}_{c_1}$ and $\chi^{||}_{c_1}$ (1^{++})



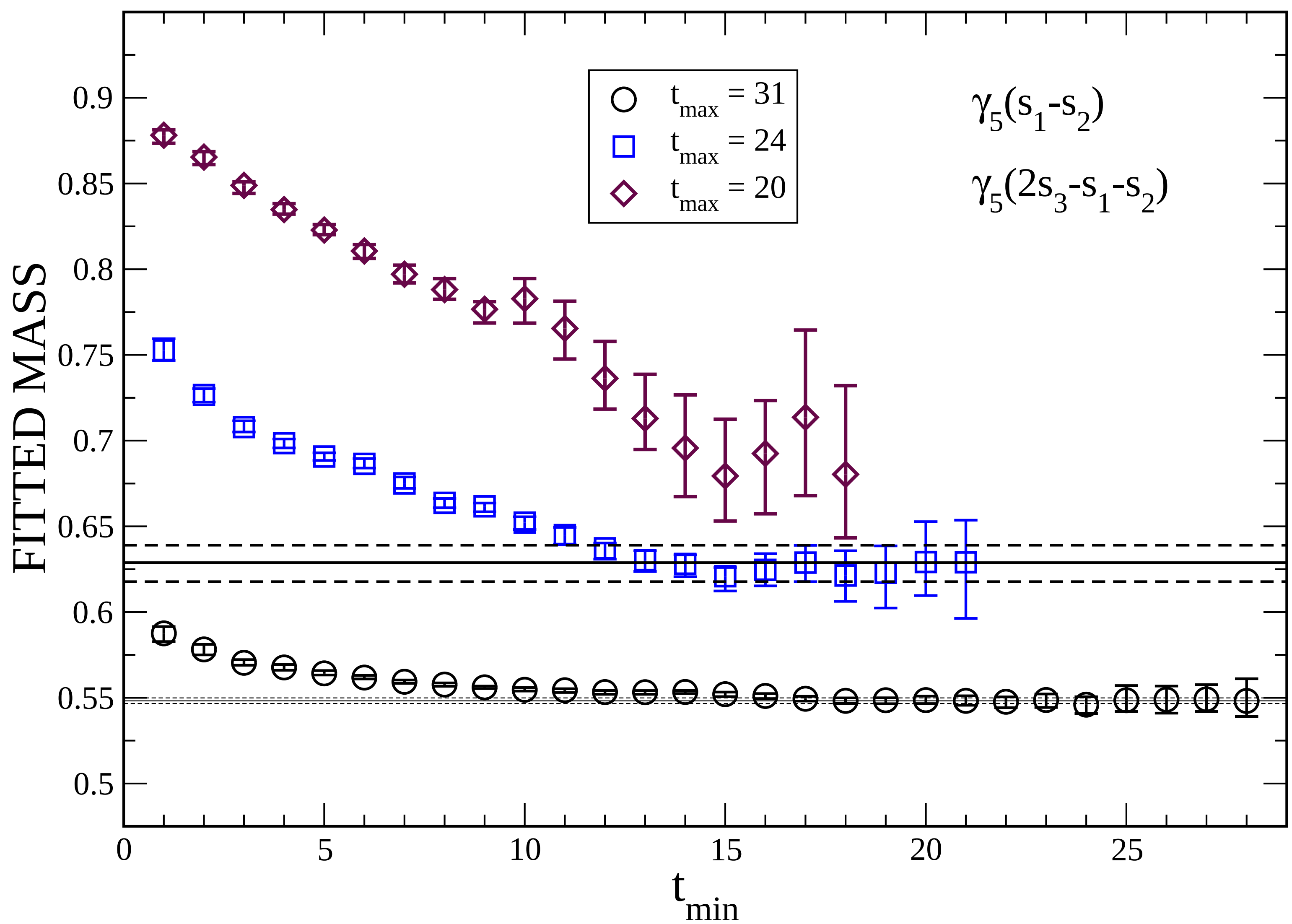
TrinLat @ TCD.

Sliding window plots for χ_{c_2}, $\chi^{|}_{c_2}$ and $\chi^{\|}_{c_2}$ (2^{++})



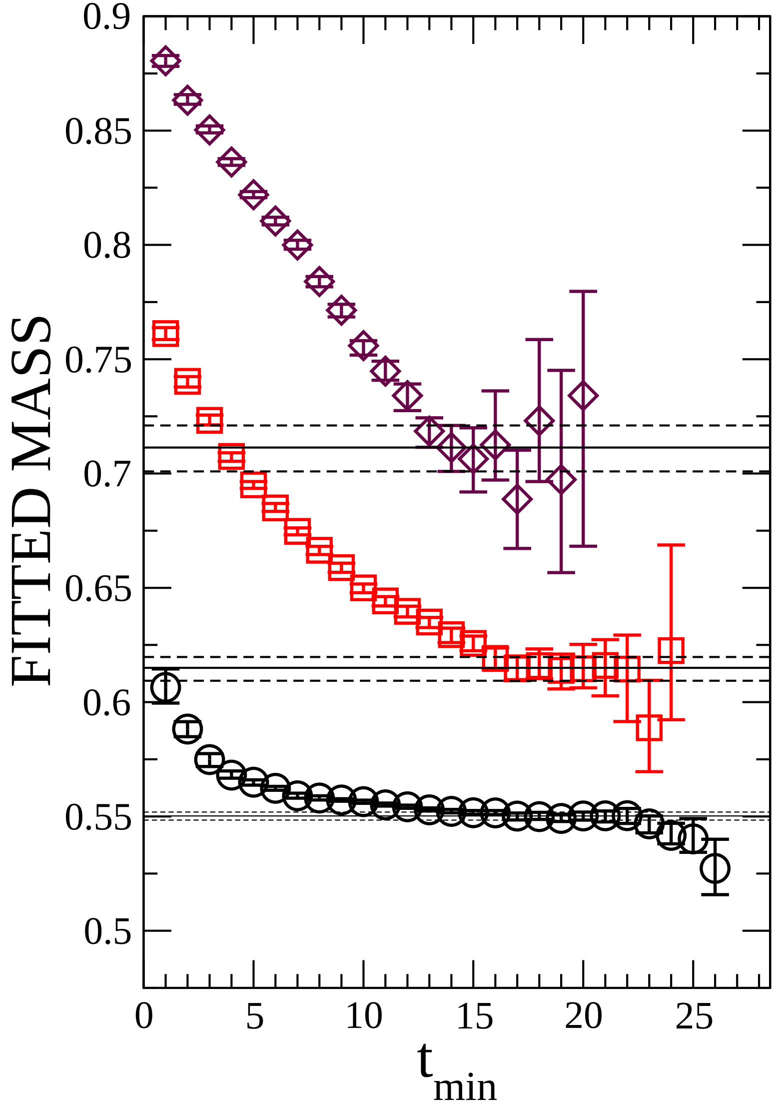
TrinLat @ TCD.

Sliding window plots for $^1D_2$ and $^1D'_2$ (2^{-+})

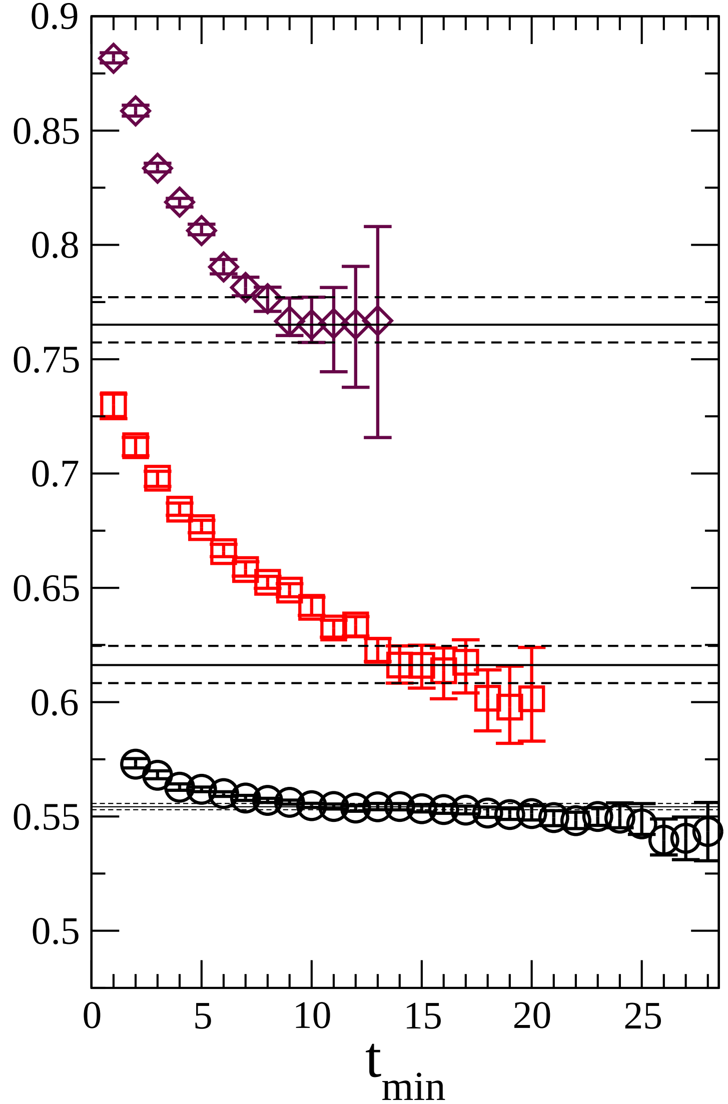


TrinLat @ TCD.

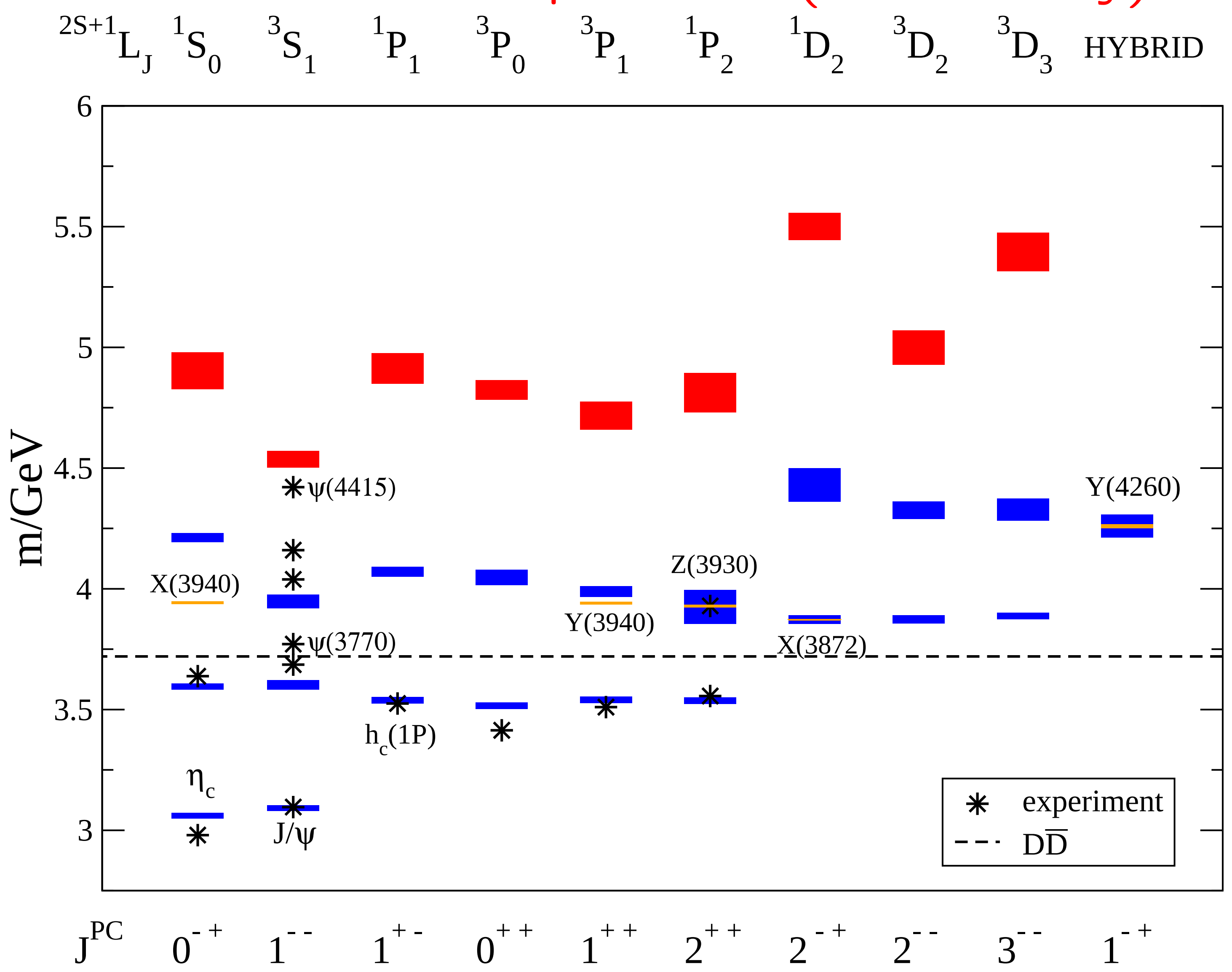
Sliding window plots for $^3D_2$ (2^{--})



Sliding window plots for $^3D_3$ (3^{--})



Charmonium Spectrum (Preliminary)

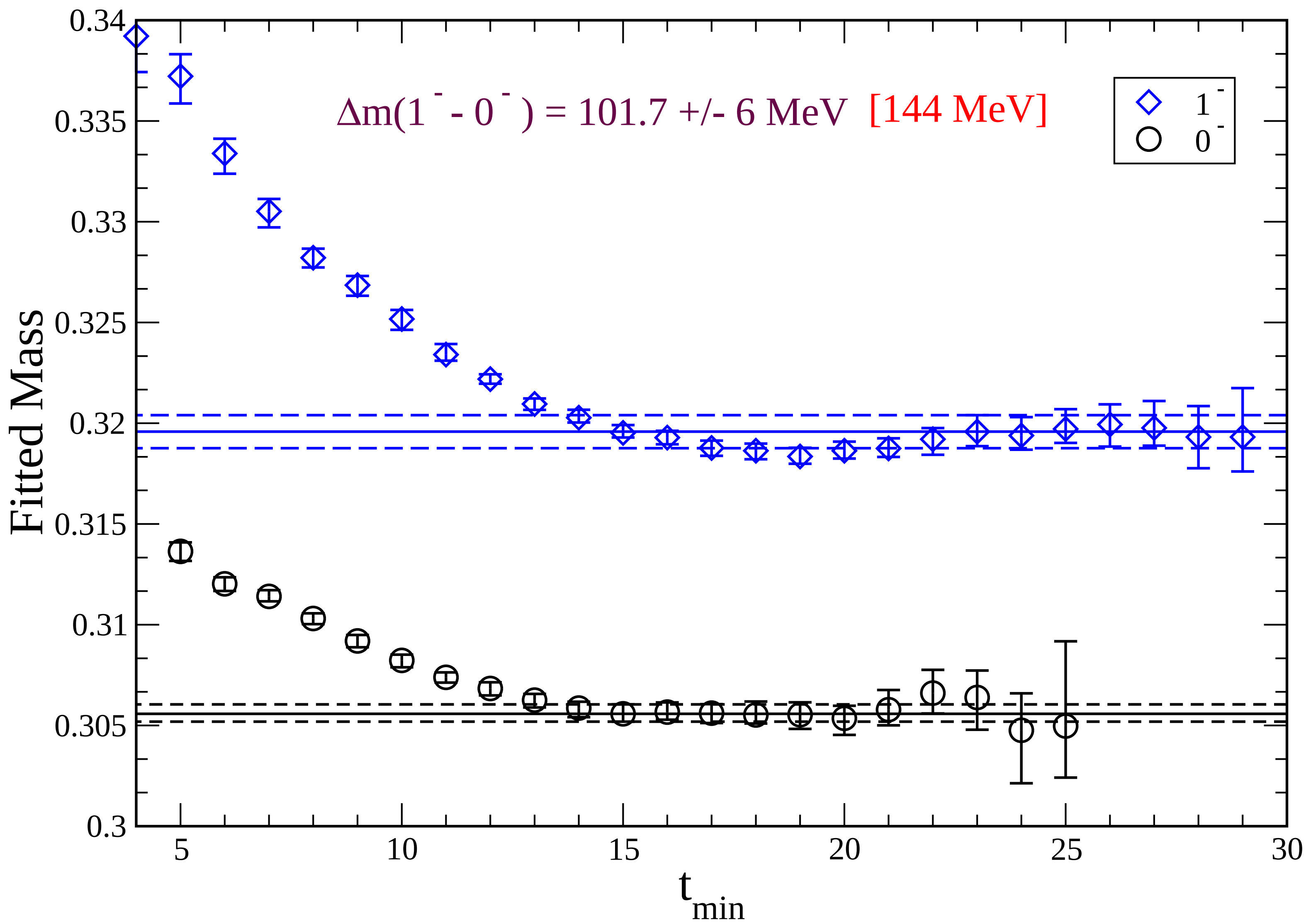


J^P	STATE	OPERATORS
0^-	D_s	$\gamma_5, \gamma_5 \sum_i s_i, \gamma_5\gamma_4$
1^-	$D_s^\star$	$\gamma_j, \gamma_i\gamma_4, \gamma_i \sum_i s_i$
0^+	$D_{s0}^\star$	$1, \gamma_4, \vec{\gamma} \cdot \vec{p}$
1^+	D_{s1}	$\gamma_5\gamma_i, \gamma_i\gamma_j,\ \vec{\gamma} \times \vec{p}, \gamma_5 p_i$
2^+	D_{s2}	$\gamma_k p_i + \gamma_i p_k,\ \gamma_1 p_1 - \gamma_2 p_2,\ 2\gamma_3 p_3 - \gamma_1 p_1 - \gamma_2 p_2$

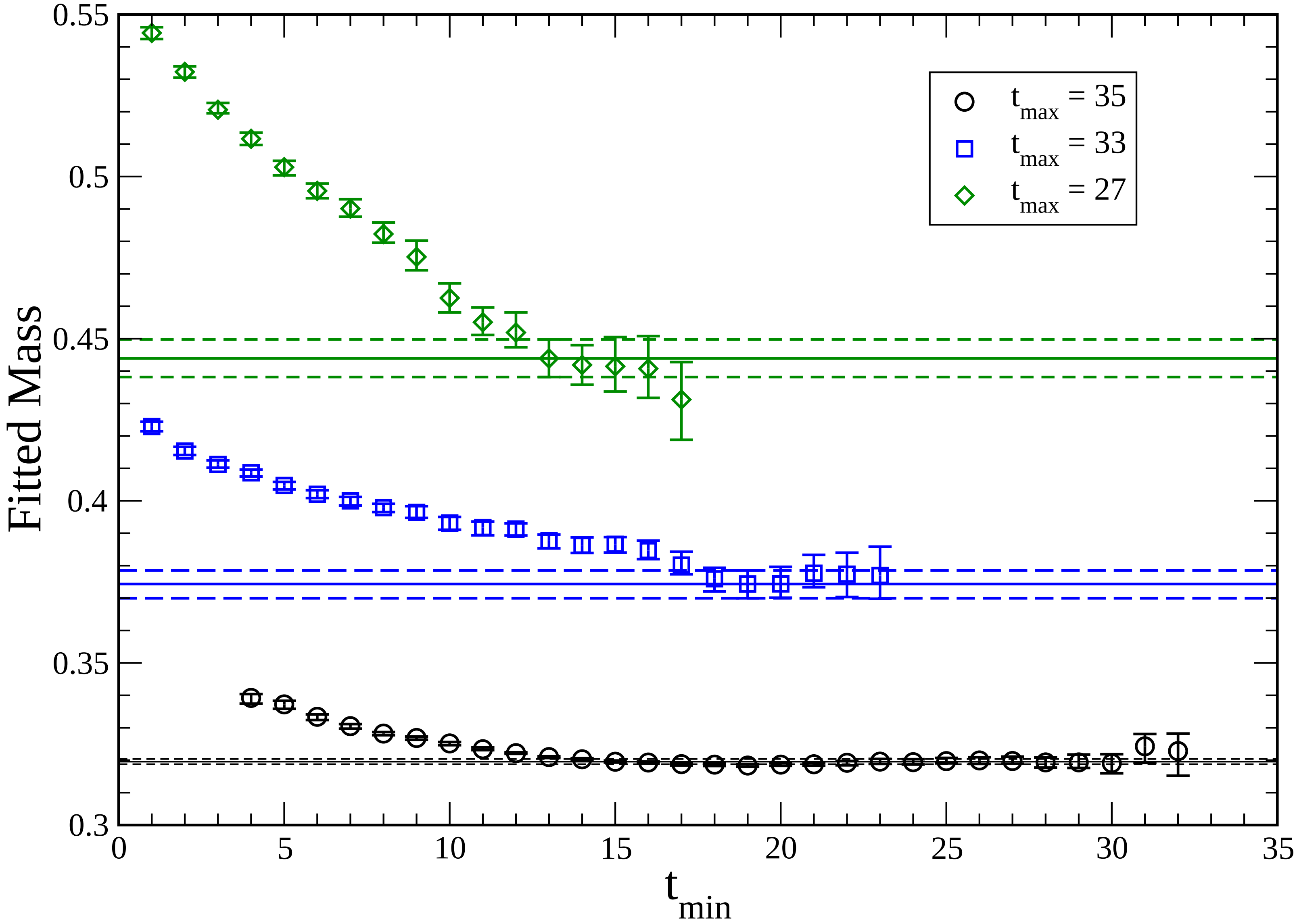
[P. Lacock, C. Michael, P. Boyle and P. Rowland, Phys. Rev. D54, 6997 (1996)]

TrinLat @ TCD.

Sliding window plots for D_s 0^- and 1^- ground states.

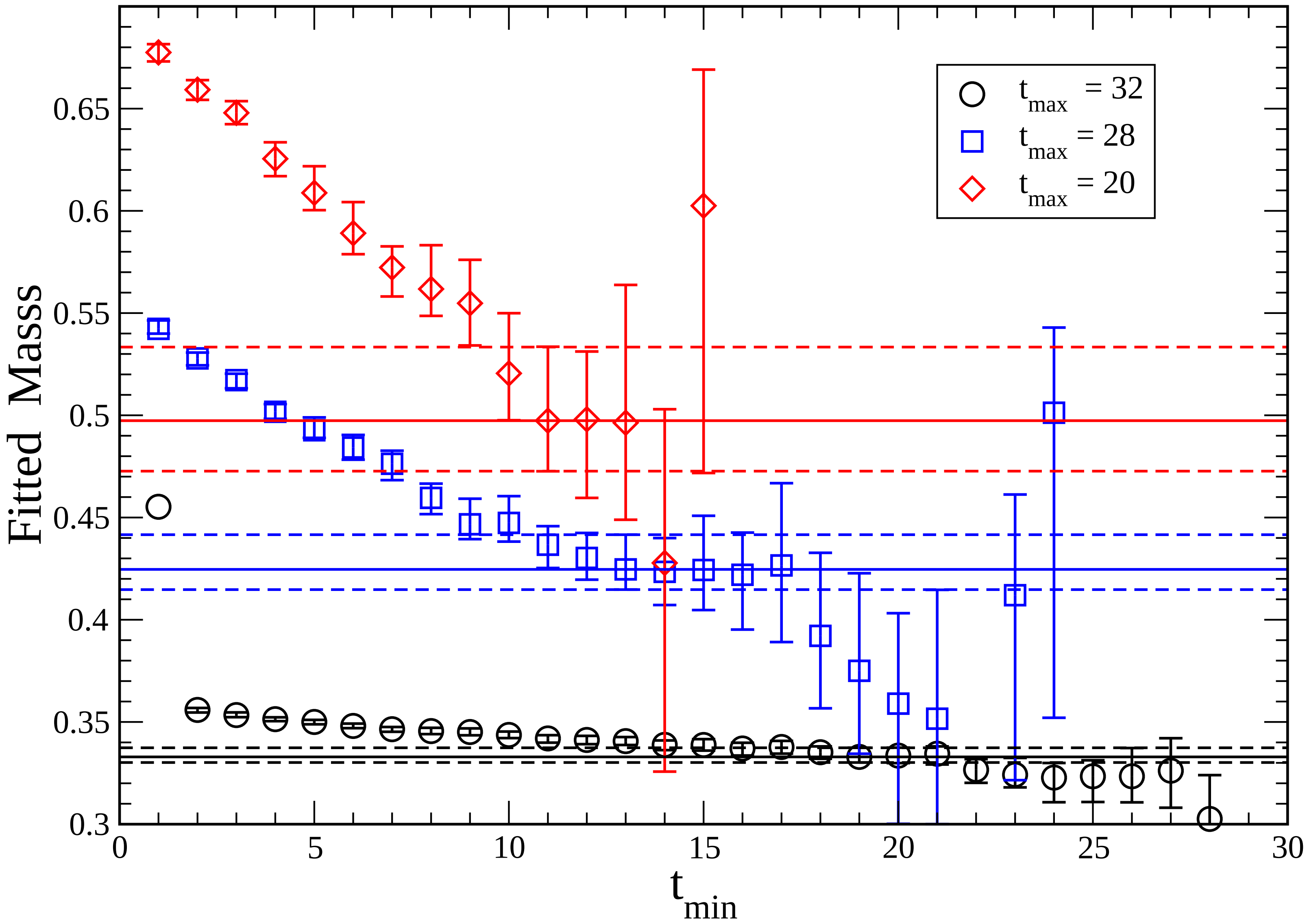


Sliding window plots for the $D_s\ 1^-$ state

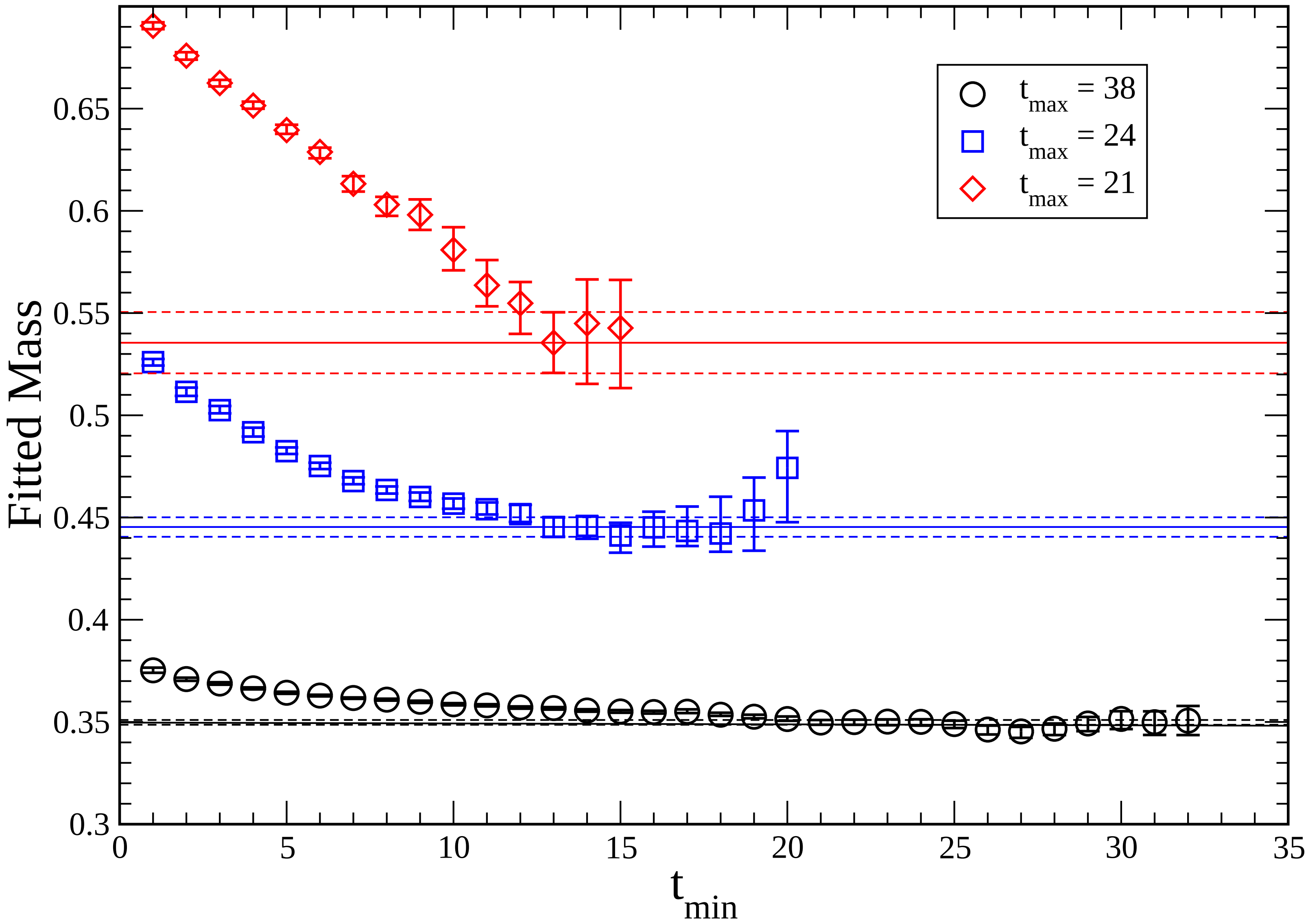


TrinLat @ TCD.

Sliding window plots for $D_s\ 0^+$ states.



Sliding window plots for D_s 1^+ state.



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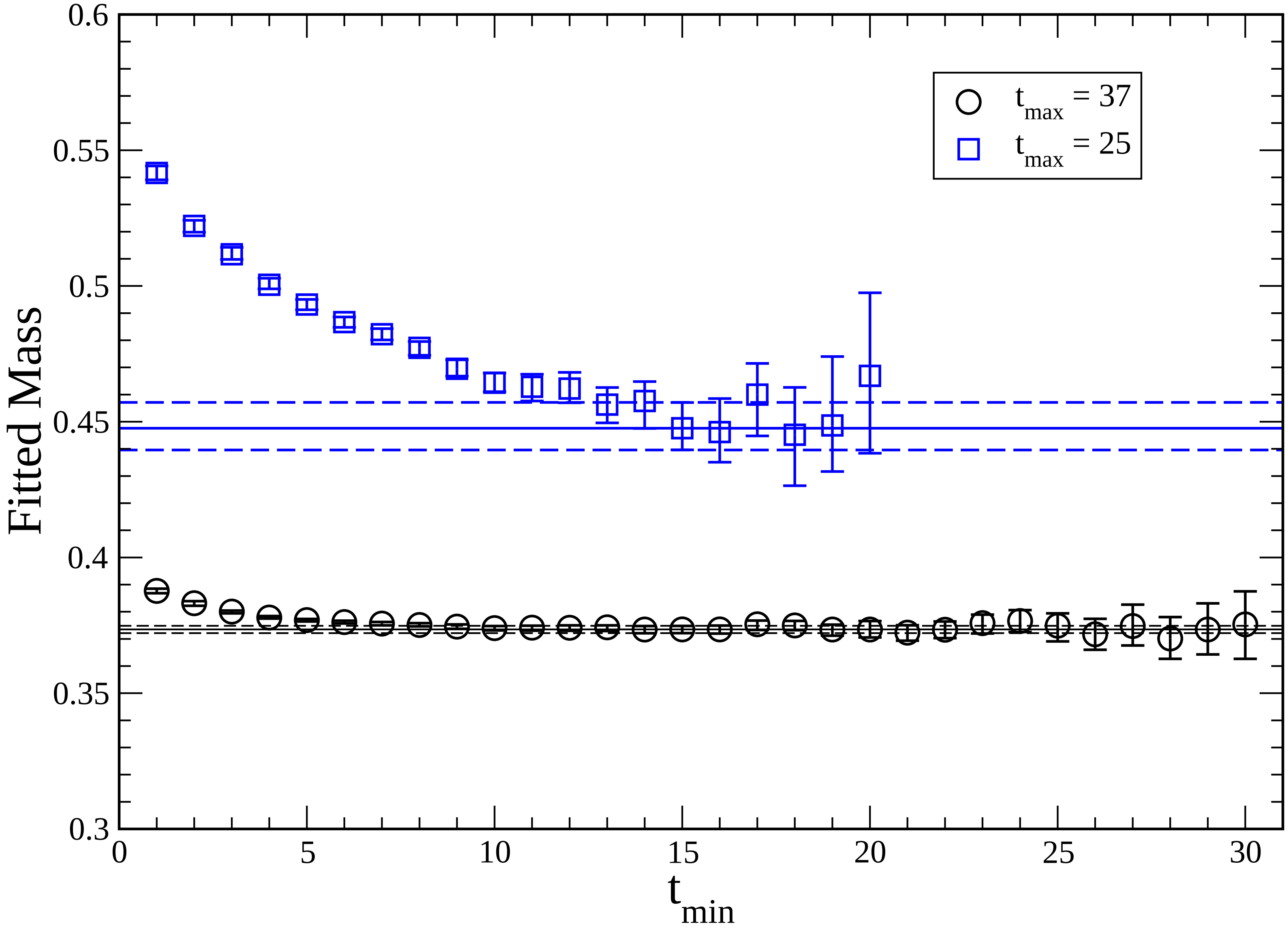
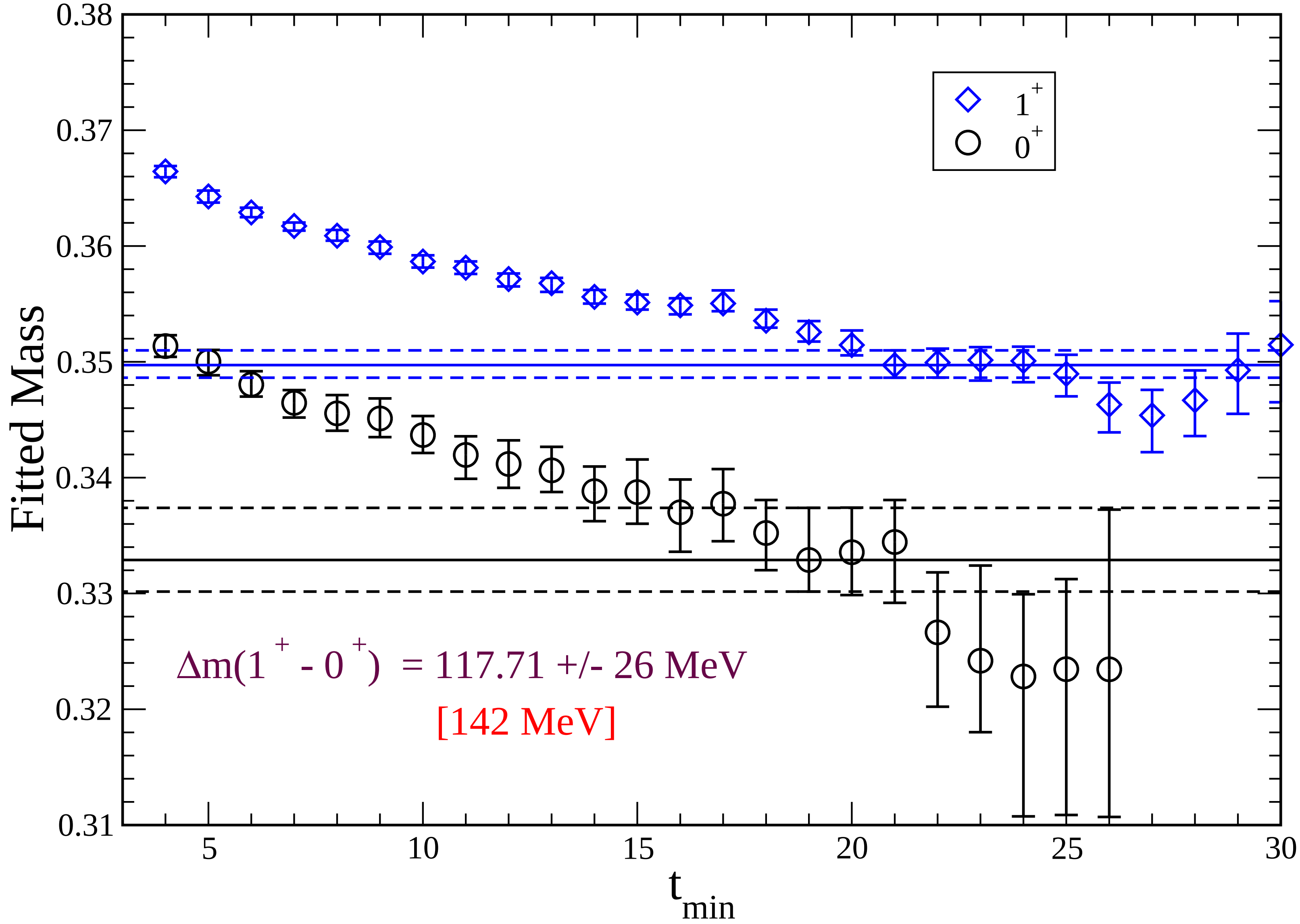
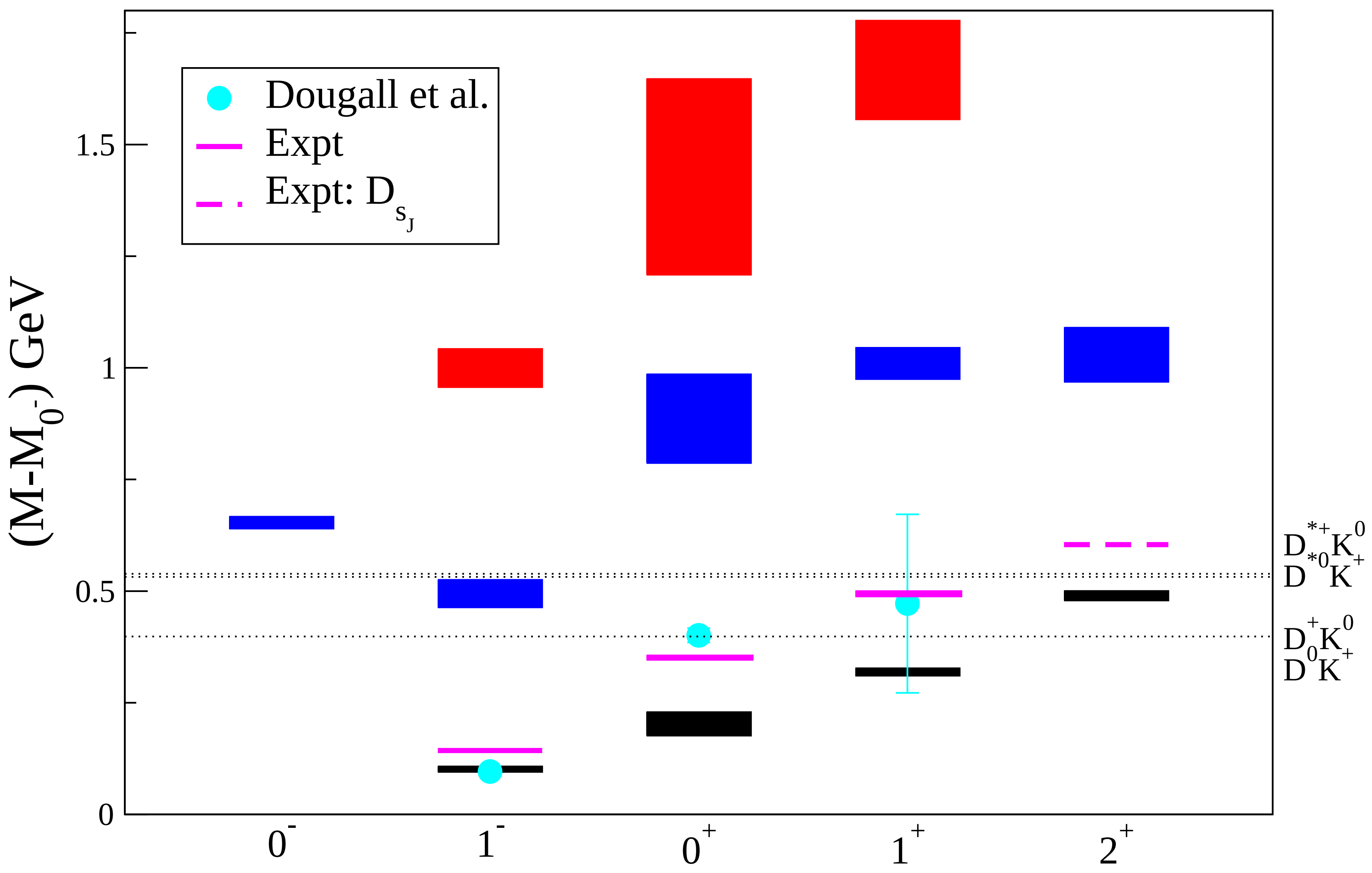
Sliding window analysis for $D_s\ 2^+$ state



Sliding window plots for D_s 0^+ and 1^+ states.



D_s Spectrum (preliminary)



TrinLat @ TCD.

Conclusions and Outlook

▷ We present results for the charmonium and the D_s spectra. All-to-all propagators are crucial. We obtain good signals for S, P and D waves at almost tuned m_c, including radial excitations. We have a good signal for the 1^{-+} hybrid in the charmonium system. We will extend our studies to other hybrids.

▷ Our results from the variational analysis are preliminary. Multi-correlator - Multi-exponential fits are being performed.

▷ Hyperfine splitting is very small in the charmonium system ~ 30 MeV .This will be addressed by including higher order correction terms ; $\Sigma \cdot B$.We will also increase the dilution level for the D_s system in order to obtain better signals and to investigate the effects of the disconnected diagrams in the $c\bar{c}$ system.

▷ We also studied the charmonium at high temperatures [arXiv:0705.2198] .