Improved Factorization Method in Studying B-meson Decays

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Abstract

B decays are a subject of active research since they provide useful information on the dynamics of strong and electroweak interactions for testing the Standard Model (SM) and models beyond and are ideally suited for a critical analysis of CP violation phenomena. Within the standard model, there exist certain relations between CP violating rate differences in B decays in the $SU(3)$ limit, as for example $\Delta(\bar{B}^0 \to \pi^+\pi^-) = -\Delta(\bar{B}^0 \to \pi^+K^-)$. The goal of this letter is to study the direct CP violation asymmetry in a class of processes where there has been recent theoretical progress, as for example the B decays into two light pseudoscalars mesons and into a light pseudoscalar and a light vector meson. We identify relations between rate asymmetries which are valid in the SU(3) limit in the standard model and we compute SU(3) breaking corrections to them, going beyond the naive factorization by using the QCD improved factorization model of Beneke *et al.*. Finally, in some processes as for example $BR(B^- \to \eta' K^-)$, we claim that one has to add SUSY contributions to the Wilson coefficients. In these cases, we end with a BR depending on three parameters, whose values are constrained by the experimental data.

1 Introduction

In the Standard Model, CP violation arises solely from the phase in the 3×3 unitary CKM matrix, $V_{CKM} = (V_{ij})$, and any CP violating observable is proportional to $Im(V_{ij}V_{il}^*V_{kj}^*V_{kl})$, with $i \neq k$ and $j \neq l$. One can write down the SU(3) invariant amplitude for $B \to PP$ and $B \to PV$ decays in terms of the tree and penguin contributions as for example:

$$\begin{aligned} A(\bar{B}^0 \to \pi^+\pi^-) &= V_{ub}V_{ud}^*\,T + V_{cb}V_{cd}^*\,P \\ A(\bar{B}^0 \to \pi^+K^-) &= V_{ub}V_{us}^*\,T + V_{cb}V_{cs}^*\,P \end{aligned}$$

Here, in SU(3) limit, neglecting annihilation diagrams, T and P are the same in the two processes. Even there are no simple relations among the branching ratios of these decays since the CKM factors in the T and P amplitudes are different, because $\Delta_{PP}^{B} \sim Im(TP^*)Im(V_{ub}V_{uq}^*V_{cb}^*V_{cq})$ and $Im(V_{ub}V_{ud}^*V_{cb}^*V_{cd}) = -Im(V_{ub}V_{us}^*V_{cb}^*V_{cs})$ from the unitarity of the CKM matrix, one has the following relations among the CP violating rate differences [6]:

$$\Delta_{\pi^+\pi^-}^{\bar{B}^0} = -\Delta_{K^+K^-}^{\bar{B}_s^0} = \Delta_{K^+\pi^-}^{\bar{B}_s^0} = -\Delta_{\pi^+K^-}^{\bar{B}^0}\,, \tag{1}$$

and similarly for $B \to PV$, where

$$\Delta_{PP}^{B} = \Gamma(B \to PP) - \bar{\Gamma}(\bar{B} \to \bar{P}\bar{P})$$

Also, the CP asymmetry is defined as

$$A_{CP} = \frac{\Gamma(B \to PP) - \bar{\Gamma}(\bar{B} \to \bar{P}\bar{P})}{\Gamma(B \to PP) + \bar{\Gamma}(\bar{B} \to \bar{P}\bar{P})}$$

The most important question now is to establish to what precision these relations hold within the standard model, or equivalently to estimate the corrections they might receive from different sources, as for example the annihilation contributions and SU(3)breaking effects.

In this respect, we first review corrections to these relations in **Naive Factorization** and then discuss the corrections in **QCD improved factorization method** developed by Beneke *et al.*.

2 Naive Factorization

The effective weak Hamiltonian for $B \to PP$ decays is [1]

$$H_{eff} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left[C_1 O_1^p + C_2 O_2^p + \sum_{i=3}^{10} C_i O_i + C_{7\gamma} O_{7\gamma} + C_{8g} O_{8g} \right] + h.c. \quad (2)$$

where $\lambda_p = V_{pb} V_{pq}^*$, with $p = u, c$ and $q = d$ for $\Delta S = 0$ processes and $q = s$ for $\Delta S = 1$ processes. Also, one has the unitarity relation $-\lambda_t = \lambda_u + \lambda_c$.

Since in our case $a_1^c = a_2^c = 0$, the Hamiltonian (1) becomes

$$H_{eff} = \frac{G_F}{\sqrt{2}} \left[\lambda_u \left(C_1 O_1^u + C_2 O_2^u \right) + \lambda_p \left(\sum_{i=3}^{10} C_i O_i^p + C_{7\gamma} O_{7\gamma} + C_{8g} O_{8g} \right) \right] + h.c. \quad (3)$$

being expressed in terms of the Wilson coefficients C_i (evaluated, in our case, at the renormalization scale $\mu = m_B$) and the following operators:

- the tree level left-handed current-current operators:

$$O_1^u = (\bar{u}b)_{V-A} (\bar{q}u)_{V-A} \;, \quad O_2^u = (\bar{u}_\beta b_\alpha)_{V-A} (\bar{q}_\alpha u_\beta)_{V-A} \quad (4)$$

- the QCD penguin operators

$$O_3 = (\bar{q}b)_{V-A} \sum_{q'} (\bar{q}'q')_{V-A} \quad , \quad O_4 = (\bar{q}_\beta b_\alpha)_{V-A} \sum_{q'} \left(\bar{q}'_\alpha q'_\beta \right)_{V-A}$$
$$O_5 = (\bar{q}b)_{V-A} \sum_{q'} (\bar{q}'q')_{V+A} \quad , \quad O_6 = (\bar{q}_\beta b_\alpha)_{V-A} \sum_{q'} \left(\bar{q}'_\alpha q'_\beta \right)_{V+A} \quad (5)$$

- the electroweak penguin operators (neutral currents coupled to Z and γ and box diagrams)

$$O_7 = (\bar{q}b)_{V-A} \sum_{q'} \left(\frac{3}{2} e_{q'} \bar{q}' q' \right)_{V+A} \quad , \quad O_8 = (\bar{q}_\beta b_\alpha)_{V-A} \sum_{q'} \left(\frac{3}{2} e_{q'} \bar{q}'_\alpha q'_\beta \right)_{V+A}$$
$$O_9 = (\bar{q}b)_{V-A} \sum_{q'} \left(\frac{3}{2} e_{q'} \bar{q}' q' \right)_{V-A} \quad , \quad O_{10} = (\bar{q}_\beta b_\alpha)_{V-A} \sum_{q'} \left(\frac{3}{2} e_{q'} \bar{q}'_\alpha q'_\beta \right)_{V-A} \quad (6)$$

  where q' runs over the active quarks at the scale $\mu = m_B$, $q' = u, d, s, c, b$, and e_q are the corresponding electric charges, in units of $|e|$,

- the electromagnetic and chromomagnetic dipole operators

$$\begin{aligned} O_{7\gamma} &= (-e/8\pi^2) m_b \bar{q} \sigma^{\mu\nu} (1+\gamma_5) b F_{\mu\nu} \\ O_{8g} &= (-g_s/8\pi^2) m_b \bar{q}_\alpha \sigma^{\mu\nu} (1+\gamma_5) (\lambda^A_{\alpha\beta}/2) b_\beta G^A_{\mu\nu} \end{aligned} \tag{7}$$

We apply *naive factorization* and write the matrix element of the Hamiltonian

$$\langle P_1 P_2 | H_{eff} | B \rangle = \mathcal{C} \langle P_2 | j^\mu L | 0 \rangle \langle P_1 | j_\mu L | B \rangle + (1 \leftrightarrow 2)\,, \; L = 1 - \gamma_5\,, \tag{8}$$

in terms of the form factors

$$\langle P_1(p_1) | j_\mu L | B(p_B) \rangle = \left[(p_B + p_1)_\mu - \frac{m_B^2 - m_1^2}{q^2} q_\mu \right] F_1(q^2) + \frac{m_B^2 - m_1^2}{q^2}\, q_\mu F_0(q^2) \tag{9}$$

and decay constant of the meson which is factorized

$$\langle P_2(q) | j_\mu L | 0 \rangle = i f_{P_2} q_\mu\,, \tag{10}$$

where $q = p_B - p_1$. Now, (8) becomes

$$\langle P_1 P_2 | H_{eff} | B \rangle = i \frac{G_F}{\sqrt{2}} \lambda_p \left(\frac{1}{N_c} C_i + C_j \right) f_{P_2} (m_B^2 - m_1^2) F_0^{B \to P_1}(m_2^2) + (1 \leftrightarrow 2) \tag{11}$$

where $N_c = 3$ is the number of colors and we introduce the usual combinations of the Wilson coefficients

$$a_i \equiv C_i + \frac{1}{3} C_{i+1} \;(\text{for } i = odd)\;;\; a_i \equiv C_i + \frac{1}{3} C_{i-1} \;(\text{for } i = even) \tag{12}$$

The matrix elements for the $\bar{B}_0 \to \pi^+\pi^-$ and $\bar{B}_0 \to \pi^+ K^-$ respectively are [1]:

$$\begin{aligned} A(\bar{B}_0 \to \pi^+\pi^-) &= -i \frac{G_f}{\sqrt{2}} f_\pi F_0^{B\to\pi}(m_\pi^2)(m_B^2 - m_\pi^2) \\ &\times \; \{ v_u^d\, a_1 - v_t^d \left[a_4 + a_{10} + r_\pi (a_6 + a_8) \right] \} \\ A(\bar{B}_0 \to \pi^+ K-) &= -i \frac{G_f}{\sqrt{2}} f_K F_0^{B\to\pi}(m_K^2)(m_B^2 - m_\pi^2) \\ &\times \; \{ v_u^s\, a_1 - v_t^s \left[a_4 + a_{10} + r_K (a_6 + a_8) \right] \} \end{aligned} \tag{13}$$

where

$$v_u^d = V_{ub}V_{ud}^*\,,\ v_c^d = V_{cb}V_{cd}^*\,,\ v_u^s = V_{ub}V_{us}^*\,,\ v_c^s = V_{cb}V_{cs}^* \tag{14}$$

and

$$r_{\pi(K)} = \frac{2m_{\pi(K)}^2}{(m_b - m_u)(m_u + m_{d(s)})} \approx 1.2 \tag{15}$$

Using the unitarity relations $-v_t^d = v_u^d + v_c^d$ and $-v_t^s = v_u^s + v_c^s$ and introducing the notations:

$$\begin{aligned}
&\frac{v_u^d}{v_c^d} \equiv -\,R_d\, e^{-i\gamma}\,,\quad \frac{v_u^s}{v_c^s} \equiv R_s\, e^{-i\gamma} \\
&\alpha \equiv a_4 + a_{10} + r(a_6 + a_8)\,,\quad \beta \equiv a_1 + \alpha
\end{aligned} \tag{16}$$

the amplitudes (13) can be put in a simplified form

$$\begin{aligned}
A(\bar{B}_0 \to \pi^+\pi^-) &= -\,i\frac{G_f}{\sqrt{2}} f_\pi F_0^{B\to\pi}(m_B^2 - m_\pi^2) v_c^d \left[-R_d\, e^{-i\gamma}\beta + \alpha\right] \\
A(\bar{B}_0 \to \pi^+K^-) &= -\,i\frac{G_f}{\sqrt{2}} f_K F_0^{B\to\pi}(m_B^2 - m_\pi^2) v_c^s \left[R_s\, e^{-i\gamma}\beta + \alpha\right]
\end{aligned} \tag{17}$$

which allows us to write down the CP violating amplitude difference

$$\begin{aligned}
\left(|A|^2 - |\bar{A}|^2\right)_{B\to\pi\pi} &= \frac{G_F^2}{2} f_\pi^2 (F_0^{B\to\pi})^2 (m_B^2 - m_\pi^2)^2 4 (v_c^d)^2 R_d \sin\gamma\ \delta_{\pi\pi} \\
\left(|A|^2 - |\bar{A}|^2\right)_{B\to\pi K} &= -\,\frac{G_F^2}{2} f_K^2 (F_0^{B\to\pi})^2 (m_B^2 - m_\pi^2)^2 4 (v_c^s)^2 R_s \sin\gamma\ \delta_{\pi K}
\end{aligned} \tag{18}$$

where

$$\delta \;=\; Re(\beta) Im(\alpha) - Im(\beta) Re(\alpha) \tag{19}$$

is the same, $\delta_{\pi\pi} = \delta_{\pi K}$.

Within SU(3) flavor symmetry, when small annihilation contributions and phase space differences are neglected, naive factorization yields the relation [8]

$$\Delta_{\pi^+\pi^-}^{\bar{B}^0} = -\frac{f_\pi^2}{f_K^2} \Delta_{\pi^+K^-}^{\bar{B}^0} \tag{20}$$

which can be used to test the SM CP violation, or to predict one rate difference if the other one is known.

In what it concerns the CP asymmetries related by

$$A_{CP}(\pi^+\pi^-) = -\frac{f_\pi^2}{f_K^2}\frac{Br(\pi^+K^-)}{Br(\pi^+\pi^-)}\, A_{CP}(\pi^+K^-), \tag{21}$$

for the reported CP branching ratios [Babar]

$$Br(\pi^+\pi^-) = (5.8 \pm 0.4 \pm 0.3) \times 10^{-6}\,, \quad Br(\pi^+K^-) = (19.4 \pm 0.6) \times 10^{-6}\,,$$

the relation (21) explicitly is

$$A_{CP}(\pi^+\pi^-) \approx -2.2\, A_{CP}(\pi^+K^-)$$

Data on these asymmetries are just emerging from Babar:

$$A_{CP}(\pi^+K^-) = -0.107 \pm 0.018$$

and Belle,

$$A_{CP}(\pi^+\pi^-) = 0.55 \pm 0.08 \pm 0.05\,,$$

and one may notice that these values do not satisfy the above theoretical prediction.

3 Improved Factorization Method

We point out some features of the IFM developed by Beneke et al. [3]

- it is a systematic and model-independent calculation of two-body hadronic decays, in the heavy-quark limit;
- it includes three classes of power corrections: from operators with (pseudo-)scalar currents (as in O_6 and O_8), from the wave function distribution amplitudes and from annihilation contributions;
- it predicts branching ratio for $B \to \pi\pi$ and $B \to \pi K$ decays in good agreement with experiment;
- it puts constrains in the (ρ, η) plane in the determination of $\gamma = arg(V_{ub}^*)$;
- it applies for $B \to M_1 M_2$, when M_2 (the emission particle or the meson that does not pick up the spectator from B) is a light meson;
- it is applicable for both M_1 a light or a heavy meson (the D meson);
- the method can be applied for all $B \to PP$ decays, as well as for $B \to PV$ decays;
- the factorization formula (7) is applicable, the nonfactorizable corrections are included in the a_i parameters;
- the amplitudes are expressed in terms of the a_i and b_i (annihilation) parameters and of the operators (3-6). The a_i coefficients have imaginary parts coming from vertex corrections and penguin contributions. However, the strong phases are small, at order of α_s or Λ_{QCD}/m_b. Also, the annihilation contribution are small, being suppressed by a power of Λ_{QCD}/m_b;
- the Wilson coefficients are calculated at the scale $\mu = m_b$ using next-to-leading order modified scheme (two-loop expression for $\alpha_s(\mu)$);
- the electroweak penguin contributions are considered as next-to-leading order since they are already proportional to α. But they are of the same magnitude as the CKM-suppressed tree terms;

- there is also a contribution coming from the hard scattering with the spectator, which is missing in the naive factorization. However, these "nonfactorizable" contributions are small compared to the leading ones (except again for the color supressed amplitudes);
- there are uncertainties coming from the CKM matrix elements, form factors and endpoint divergencies.

The IFM formula when both M_1 and M_2 are light mesons is

$$\begin{aligned} \langle M_1 M_2 | O_i | B \rangle &= F_0^{B\to M_1} f_{M_2} \int dx\, T^I_{M_2,i} \phi_{M_2}(x) \;+\; (1 \leftrightarrow 2) \\ &+\; f_B f_{M_1} f_{M_2} \int dz\, dy\, dx\, T_i^{II}(x,y,z) \phi_B(z) \phi_{M_1}(y) \phi_{M_2}(x) \end{aligned} \quad (22)$$

where

- ϕ are the leading-twist light-cone distribution amplitudes and the integration is over the momentum fractions inside the mesons
- $T_i^{I,II}$ are the hard-scattering kernels at next-to-leading order (NLO) in α_s :
  - T_i^I includes tree diagrams plus corrections (hard gluon exchanges and penguins) and
  - T_i^{II} expresses the hard gluon exchange with the spectator (Figure 1).

  For $T_i^I = 1$ and $T_i^{II} = 0$, we recover the naive factorization.

Each diagram in Figure 2 contains a leading-power contribution.

The meson wave functions will be an important source of SU(3) breaking. For the light mesons, we have twist-2 and twist-3 distribution amplitudes respectively defined in the following bilocal operator matrix elements:

$$\begin{aligned} \langle M(p) | \bar{q}(z_2) \gamma_\mu \gamma_5 q(z_1) | 0 \rangle &= -\, f_M p_\mu \int_0^1 dx\; e^{i(xp\cdot z_2 + \bar{x} p \cdot z_1)} \phi(x) \\ \langle M(p) | \bar{q}(z_2) i \gamma_5 q(z_1) | 0 \rangle &= f_M \mu_M \int_0^1 dx\; e^{i(xp\cdot z_2 + \bar{x} p \cdot z_1)} \phi_p(x)\,, \end{aligned} \quad (23)$$

where μ_M is expressed in terms of the quark masses as $\mu_M = m_M^2/(m_1+m_2)$ and $\bar{x} = 1 - x$. In the momentum space, the light-cone projector operator of a light pseudoscalar meson described by both the twist-2 and twist-3 amplitudes is:

$$\Phi(M) = \frac{if_M}{4N_c}\left\{\hat{p}\gamma_5\phi(x) - \mu_M\,\gamma_5\phi_p(x)\right\}, \text{ where } \hat{p} = p\cdot\gamma \tag{24}$$

We notice that in a_1, a_2, a_3, a_4, a_5, a_7, a_9, a_{10}, where we have $(V-A)(V\pm A)$, only the twist-2 amplitude is taken, while in a_6, a_8 (the terms proportional with $r = 2\mu/m_b$) only the twist-3 amplitude must be considered. The operators a_6 and a_8 are important for penguin-dominant B decays.

The twist-2 distribution amplitude, $\phi(x)$, has the following expansion in Gegenbauer polynomials [2]

$$\phi(x) = 6x(1-x)[1 + \alpha_1 C_1^{(3/2)}(2x-1) + \alpha_2 C_2^{3/2}(2x-1) + ...], \tag{25}$$

with $C_1^{3/2}(u) = 3u$ and $C_2^{3/2}(u) = (3/2)(5u^2-1)$, and is different for π and K. For π, the distribution in x must be even because the u and d quarks have negligible masses and their distributions inside the pion are symmetric. This dictates $\alpha_1^\pi = 0$. The coefficient α_2^π is estimated to be 0.1 ± 0.3. For K, the u (or d) and s quarks inside the kaon are different, leading to an asymmetry in the x distribution. So a non-zero value for α_1^K is needed and it is estimated to be 0.3 ± 0.3, while $\alpha_2^K = 0.1\pm0.3$. The leading twist distribution amplitudes, valid for $\mu\to\infty$, are $\phi(x) = 6x(1-x)$ and $\phi_p(x) = 1$.

The B meson distribution between the heavy quark and light antiquark is described by

$$\phi_B = \mathcal{N}\frac{z^2(1-z)^2}{(a^2(1-z)+z^2)^2} \tag{26}$$

where the parameter a is related to the position of the maximum of the amplitude and is very small $a \in [0.05 \div 0.1]$. However, since the momentum is almost carried by the heavy quark, one may work with a strongly peaked function around $z_0 = \lambda_B/m_B \approx 0.066 \pm 0.029$, for $\lambda_B = 0.35 \pm 0.15$ GeV.

4 The a_i coefficients in IFM

4.1 Vertex correction

We consider the diagrams (a), (b), (c) and (d) in Figure 2, and calculate the one gluon exchange contribution to the O_2 operator as [9]

$$\begin{aligned}
\langle O_2\rangle_V &\sim g_s^2 \frac{f_{M_2}}{4}\frac{C_F}{N}\int_0^1 dx\,\phi_{M_2}(x)\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2}\cdots \mathrm{Tr}\,[\cdots] \\
&= \frac{\alpha_s}{4\pi}\frac{C_F}{N}\langle M_2|j^\mu L|0\rangle\langle M_1|j_\mu L|B\rangle \int_0^1 dx\,\phi_{M_2}(x)\left[-18-12\ln\frac{\mu}{m_b}+g(x)\right] \\
&\equiv \frac{\alpha_s}{4\pi}\frac{C_F}{N}\langle M_2|j^\mu L|0\rangle\langle M_1|j_\mu L|B\rangle\, V_{M_2} \qquad (27)
\end{aligned}$$

The **vertex correction** V_{M_2} (after dimensional regularization of the infrared divergence) includes the function

$$\begin{aligned}
g(x) &= 3\left(\frac{1-2x}{1-x}\ln x - i\pi\right) \\
&+ \left[2\mathrm{Li}_2(x) - \ln^2 x + \frac{2\ln x}{1-x} - (3+2i\pi)\ln x - (x\to\bar{x})\right] \qquad (28)
\end{aligned}$$

which contains an imaginary part and simplifies to the first term in the r.h.s. for symmetric wave functions.

4.2 Hard scattering with the spectator

The diagrams (g) and (h), corresponding to the hard scattering with the spectator, lead to the following result

$$\begin{aligned}
\langle O_2\rangle_H &= -i\,g_s^2\frac{f_B f_{M_1} f_{M_2}}{4^3}\frac{C_F}{N^2}\int_0^1 dzdxdy\,\phi_B(z)\phi_{M_1}(y)\phi_{M_2}(x)\left[\mathrm{Tr}_{(g)}+\mathrm{Tr}_{(h)}\right] \\
&\equiv \frac{\alpha_s}{4\pi}\frac{C_F}{N}\langle M_2|j^\mu L|0\rangle\langle M_1|j_\mu L|B\rangle H \qquad (29)
\end{aligned}$$

where

$$H = \frac{4\pi^2}{N}\frac{f_B f_{M_1}}{F_0^{B\to M_1} m_B^2}\int_0^1 dz\,\frac{\phi_B(z)}{z}\int_0^1 dx\,\frac{\phi_{M_2}(x)}{\bar{x}}\int_0^1 dy\,\frac{\phi_{M_1}(y)}{\bar{y}} \qquad (30)$$

Even the twist-3 amplitude gives supressed contributions, one may include both twist-2 and twist-3 distribution amplitudes and, in this case, the **hard-scattering spectator** contribution becomes

$$H_{M_1 M_2} = \frac{4\pi^2}{N} \frac{f_B f_\pi}{m_B \lambda_B F_0^{B\to M_1}} \int_0^1 \frac{dx}{\bar{x}} \phi_{M_2}(x) \int_0^1 \frac{dy}{\bar{y}} \left[\phi_{M_1}(y) + \frac{2\mu}{m_b} \frac{\bar{x}}{x} \phi_p(y)\right] \tag{31}$$

For the logarithimic divergency coming from $X_H = \int_0^1 \phi_p \, dy/\bar{y} = \int_0^1 dy/\bar{y}$ which, in principle, would be absent in a full theory, we introduce an infrared cut-off at $\Lambda_h = 0.5$ GeV and use

$$X_H = \ln \frac{m_B}{\Lambda_h} \,. \tag{32}$$

The final results are insensitive to the precise value of the cut-off.

4.3 Penguin corrections

There are two types of penguin corrections [3, 9].

4.4 Penguin type diagrams of $O_1 - O_6$ operators

The penguin-type diagrams of the $O_1 - O_6$ operators are represented in Figure 2e. Inspecting the spinor indices in the expressions of these operators, we notice two type of contractions. Although these are related by Fierz transformations in 4 dimensions, they are not related in d dimensions (for dimensional regularization), leading to different results.

For the contraction in the operators O_4 and O_6 one gets the following contribution of the penguin-type diagrams of these operators (see Figure (e1))

$$\mathcal{P}_4 = \frac{\alpha_s}{4\pi} \frac{C_F}{N} \langle M_2 | j^\mu L | 0 \rangle \langle M_1 | j_\mu L | B \rangle \sum_{q'=u,\ldots,b} \left[\frac{4}{3} \ln \frac{m_b}{\mu} + G(s_{q'})\right] \tag{33}$$

and

$$\mathcal{P}_6 = \frac{\alpha_s}{4\pi} \frac{C_F}{N} \langle M_2 | j^\mu L | 0 \rangle \langle M_1 | j_\mu L | B \rangle \sum_{q'=u,\ldots,b} \left[\frac{4}{3} \ln \frac{m_b}{\mu} + \hat{G}(s_{q'})\right] \tag{34}$$

In the case of the second contraction (see O_1 and O_3 operators) there is an extra term 2/3. This appears in the contribution coming from the current-current operator O_1,

$$\mathcal{T}^p = \frac{\alpha_s}{4\pi}\frac{C_F}{N}\langle M_2|j^\mu|0\rangle\langle M_1|j_\mu|B\rangle\left[\frac{2}{3}+\frac{4}{3}\ln\frac{m_b}{\mu}+G(s_p)\right] \tag{35}$$

as well as in the one from the QCD penguin operator O_3

$$\mathcal{P}_3 = \frac{\alpha_s}{4\pi}\frac{C_F}{N}\langle M_2|j^\mu|0\rangle\langle M_1|j_\mu|B\rangle\sum_{q'=q,b}\left[\frac{2}{3}+\frac{4}{3}\ln\frac{m_b}{\mu}+G(s_p)\right] \tag{36}$$

The functions

$$\begin{aligned} G(s) &= \int_0^1 dx\, G(s-i\epsilon,\bar{x})\phi_{M_2}(x), \\ \hat{G}(s) &= \int_0^1 dx\, G(s-i\epsilon,\bar{x})\phi^p_{M_2}(x), \end{aligned} \tag{37}$$

where $s_i = m_i^2/m_b^2$ are the mass ratios of the quarks involved in the penguins, correspond to the twist-2 and twist-3 amplitudes and contain the usual function from the NDR renormalization scheme [1]

$$G(s_q) = 4\int du\, u\bar{u}\ln[s_q^2 - u\bar{u}k^2] \tag{38}$$

where k is the momentum transferred by the gluon to the $\bar{q}'q'$ pair ($k^2 \approx m_B^2/2$). Since, in our case, $k = p - xq$, the above function explicitly becomes

$$\begin{aligned} G(s,\bar{x}) &= 4\int_0^1 du\, u\bar{u}\ln[s - u\bar{u}\bar{x}] \\ &= -\frac{10}{9}+\frac{2}{3}\ln s-\frac{8s}{3\bar{x}}+\frac{4}{3}\left(1+\frac{2s}{\bar{x}}\right)\sqrt{\frac{4s}{\bar{x}}-1}\,\arctan\frac{1}{\sqrt{\frac{4s}{\bar{x}}-1}} \end{aligned} \tag{39}$$

4.5 The magnetic penguin insertion

is computed with the dipole operator O_{8g} at the tree level, and gives the following result (see Figure (2f)):

$$\langle O_{8g}\rangle = -2\frac{\alpha_s}{4\pi}\frac{C_F}{N}\langle M_2|j^\mu|0\rangle\langle M_1|j_\mu|B\rangle\int_0^1 dx\,\frac{\phi_{M_2}(x)}{\bar{x}} \tag{40}$$

and a similar expression for $\phi^p_{M_2}(x)$, but without $\bar{x}$ in the denominator.

Putting everything together, let us write down the a_i coefficients in the improved factorization approach as [3]:

$$
\begin{aligned}
a_1(M_1M_2) &= C_1 + \frac{C_2}{N_c}\left[1 + \frac{C_F\alpha_s}{4\pi}(V_{M_2}+H)\right], \\
a_2(M_1M_2) &= C_2 + \frac{C_1}{N_c}\left[1 + \frac{C_F\alpha_s}{4\pi}(V_{M_2}+H)\right], \\
a_3(M_1M_2) &= C_3 + \frac{C_4}{N_c}\left[1 + \frac{C_F\alpha_s}{4\pi}(V_{M_2}+H)\right], \\
a_4^p(M_1M_2) &= C_4 + \frac{C_3}{N_c}\left[1 + \frac{C_F\alpha_s}{4\pi}(V_{M_2}+H)\right] + \frac{C_F\alpha_s}{4\pi N_c}P^p_{M_2,2}, \\
a_5(M_1M_2) &= C_5 + \frac{C_6}{N_c}\left[1 + \frac{C_F\alpha_s}{4\pi}(-12-V_{M_2}-H)\right], \\
a_6^p(M_1M_2) &= C_6 + \frac{C_5}{N_c}\left(1 - 6\frac{C_F\alpha_s}{4\pi}\right) + \frac{C_F\alpha_s}{4\pi N_c}P^p_{M_2,3}, \\
a_7(M_1M_2) &= C_7 + \frac{C_8}{N_c}\left[1 + \frac{C_F\alpha_s}{4\pi}(-12-V_{M_2}-H)\right], \\
a_8^p(M_1M_2) &= C_8 + \frac{C_7}{N_c}\left(1 - 6\frac{C_F\alpha_s}{4\pi}\right) + \frac{\alpha}{9\pi N_c}P^{p,EW}_{M_2,3}, \\
a_9(M_1M_2) &= C_9 + \frac{C_{10}}{N_c}\left[1 + \frac{C_F\alpha_s}{4\pi}(V_{M_2}+H)\right], \\
a_{10}^p(M_1M_2) &= C_{10} + \frac{C_9}{N_c}\left[1 + \frac{C_F\alpha_s}{4\pi}(V_{M_2}+H)\right] + \frac{\alpha}{9\pi N_c}P^{p,EW}_{M_2,2},
\end{aligned}
\tag{41}
$$

where $C_F = (N_c^2-1)/2N_c$. The vertex, the hard gluon exchange with the spectator and the penguin contributions, at $\mu = m_b$, are:

$$
\begin{aligned}
V_M &= -18 + \int_0^1 dx g(x)\phi_M(x), \\
P^p_{M,2} &= C_1\left[\frac{2}{3} + G_M(s_p)\right] + C_3\left[\frac{4}{3} + G_M(0) + G_M(3)\right] \\
&+ (C_4+C_6)\left[(n_f-2)G_M(0) + G_M(s_c) + G_M(1)\right] - 2C^{eff}_{8g}\int_0^1 \frac{dx}{\bar{x}}\phi_M(x), \\
P^{p,EW}_{M,2} &= (C_1 + N_cC_2)\left[\frac{2}{3} + G_M(s_p)\right] - 3C^{eff}_{7\gamma}\int_0^1 \frac{dx}{\bar{x}}\phi_M(x),
\end{aligned}
$$

$$
\begin{aligned}
P^p_{M,3} &= C_1\left[\frac{2}{3}+\hat{G}_M(s_p)\right]+C_3\left[\frac{4}{3}+\hat{G}_M(0)+\hat{G}_M(1)\right] \\
&+ (C_4+C_6)\left[(n_f-2)\hat{G}_M(0)+\hat{G}_M(s_c)+\hat{G}_M(1)\right]-2C^{eff}_{8g}, \\
P^{p,EW}_{M,3} &= (C_1+N_cC_2)\left[\frac{2}{3}+\hat{G}_M(s_p)\right]-3C^{eff}_{7\gamma}, \\
H &= \frac{4\pi^2}{N_c}\frac{f_Bf_{M_1}}{m_B\lambda_B F_0^{B\to M_1}(0)} \\
&\times \int_0^1\frac{dx}{\bar{x}}\,\phi_{M_2}(x)\int_0^1\frac{dy}{\bar{y}}\left[\phi_{M_1}(y)+\frac{2\mu_{M_1}}{m_b}\frac{\bar{x}}{x}\phi^p_{M_1}(y)\right] \qquad (42)
\end{aligned}
$$

where the parameter $2\mu_M/m_b$ coincides with r and $s_i=m_i^2/m_b^2$ are the mass ratios for the quarks involved in the penguin diagrams, namely $s_u=s_d=s_s=0$ and $s_c=(1.3/4.2)^2$.

5 Applications

5.1 $B\to PP$ Decays

Within this approach, the amplitudes for the decays discussed in the first section are

$$
\begin{aligned}
A(\bar{B}^0\to\pi^+\pi^-) &= -i\frac{G_F}{\sqrt{2}}(m_B^2-m_\pi^2)F_0^{B\to\pi}(m_\pi^2)f_\pi[V_{ub}V^*_{ud}a_1(\pi\pi) \\
&+ V_{pb}V^*_{pd}(a^p_4(\pi\pi)+a^p_{10}(\pi\pi)+r_\pi(a^p_6(\pi\pi)+a^p_8(\pi\pi)))] \\
&- i\frac{G_F}{\sqrt{2}}f_Bf_\pi^2[V_{ub}V^*_{ud}b_1(\pi\pi) \\
&+ (V_{ub}V^*_{ud}+V_{cb}V^*_{cd})(b_3(\pi\pi)+2b_4(\pi\pi) \\
&- \frac{1}{2}(b^{EW}_3(\pi\pi)-b^{EW}_4(\pi\pi))] \\
A(\bar{B}^0\to\pi^+K^-) &= -i\frac{G_F}{\sqrt{2}}(m_B^2-m_\pi^2)F_0^{B\to\pi}(m_K^2)f_K[V_{ub}V^*_{us}a_1(\pi K) \\
&+ V_{pb}V^*_{ps}(a^p_4(\pi K)+a^p_{10}(\pi K)+r_K(a^p_6(\pi K)+a^p_8(\pi K)))] \\
&- i\frac{G_F}{\sqrt{2}}f_Bf_\pi f_K[(V_{ub}V^*_{us}+V_{cb}V^*_{cs})(b_3(\pi K)-\frac{1}{2}b^{EW}_3(\pi K)], \qquad (43)
\end{aligned}
$$

where p is summed over u and c and we have included the annihilation contributions

$$
\begin{aligned}
b_1(M_1M_2) &= \frac{C_F}{N_c^2}C_1A_1^i(M_1M_2), \\
b_3(M_1M_2) &= \frac{C_F}{N_c^2}[C_3A_1^i(M_1M_2)+C_5(A_3^i(M_1M_2)+A_3^f(M_1M_2)) \\
&+ N_cC_6A_3^f(M_1M_2)], \\
b_4(M_1M_2) &= \frac{C_F}{N_c^2}[C_4A_1^i(M_1M_2)+C_6A_2^i(M_1M_2)], \\
b_3^{EW}(M_1M_2) &= \frac{C_F}{N_c^2}[C_9A_1^i(M_1M_2)+C_7(A_3^i(M_1M_2)+A_3^f(M_1M_2)) \\
&+ N_CC_8A_3^f(M_1M_2)], \\
b_4^{EW}(M_1M_2) &= \frac{C_F}{N_c^2}[C_{10}A_1^i(M_1M_2)+C_8A_2^i(M_1M_2)], \qquad (44)
\end{aligned}
$$

with

$$
\begin{aligned}
A_j^i(M_1M_2) &= \pi\alpha_s\int_0^1 dxdy\, F_j^i(x,y)\,,\ j=\overline{1,3}\,, \\
A_3^f(M_1M_2) &= \pi\alpha_s\int_0^1 dxdy\, F_3^f(x,y)\,, \qquad (45)
\end{aligned}
$$

and

$$
\begin{aligned}
F_1^i(x,y) &= \left\{\phi_{M_2}(x)\phi_{M_1}(y)\left[\frac{1}{y(1-x\bar{y})}+\frac{1}{y\bar{x}^2}\right]+\frac{4\mu_{M_1}\mu_{M_2}}{m_b^2}\frac{2}{\bar{x}y}\right\}, \\
F_2^i(x,y) &= \left\{\phi_{M_2}(x)\phi_{M_1}(y)\left[\frac{1}{\bar{x}(1-x\bar{y})}+\frac{1}{y^2\bar{x}}\right]+\frac{4\mu_{M_1}\mu_{M_2}}{m_b^2}\frac{2}{\bar{x}y}\right\}, \\
F_3^i(x,y) &= \left\{\frac{2\mu_{M_1}}{m_b}\phi_{M_2}(x)\frac{2\bar{y}}{\bar{x}y(1-x\bar{y})}-\frac{2\mu_{M_2}}{m_b}\phi_{M_1}(y)\frac{2x}{\bar{x}y(1-x\bar{y})}\right\}, \\
F_3^f(x,y) &= \left\{\frac{2\mu_{M_1}}{m_b}\phi_{M_2}(x)\frac{2(1+\bar{x})}{\bar{x}^2y}+\frac{2\mu_{M_2}}{m_b}\phi_{M_1}(y)\frac{2(1+y)}{\bar{x}y^2}\right\} \qquad (46)
\end{aligned}
$$

We point out the following SU(3) breaking effects: the difference in the decay constants and form factors, the difference in the α_1 and α_2 coefficients that appear in the twist-2 distribution amplitude (25) and annihilation contributions. With all these taken into account, the relation (20) turns into [6]

$$\frac{\Delta^{\bar{B}^0}_{\pi^+\pi^-}}{\Delta^{\bar{B}^0}_{\pi^+K^-}} \approx -\frac{f_\pi^2}{f_K^2}\left[\frac{1-0.748\alpha_1^\pi-0.109\alpha_2^\pi-0.0013H_{\pi\pi}-0.004\delta_A^\pi}{1-0.748\alpha_1^K-0.109\alpha_2^K-0.0013H_{\pi K}+0.004\delta_A^K}\right], \quad (47)$$

where $\delta_A^\pi = 1 - 1.34X_A^\pi - 0.36(X_A^\pi)^2$ and $\delta_A^K = 0.1 - 0.8X_A^K + 1.4(X_A^K)^2$ indicate the annihilation contributions which are very small. The $H_{\pi\pi}$ and $H_{\pi K}$ are in the range between 10 and 18.

The decay amplitudes for $\bar{B}_s^0 \to K^+\pi^-$ and $\bar{B}_s^0 \to K^+K^-$ can be obtained by using the appropriate transition form factor $F_0^{B_s\to K}$ and by changing $1/m_B^2\lambda_B$ to $1/m_{B_s}^2\lambda_{B_s}$ in $H_{M_1M_2}$. One gets the same expression, (47), and thus we have come to the following relation:

$$\frac{\Delta^{\bar{B}^0}_{\pi^+\pi^-}}{\Delta^{\bar{B}^0}_{\pi^+K^-}} \approx \frac{\Delta^{\bar{B}_s^0}_{K^+\pi^-}}{\Delta^{\bar{B}_s^0}_{K^+K^-}} \quad (48)$$

which is independent of the twist-2 distribution amplitudes or meson decay constants and therefore is particularly interesting for testing the SM with less uncertainties. For example, with $\alpha_1^\pi = 0$, $\alpha_2^\pi = 0.1$, $\alpha_1^K = 0.3$, $\alpha_2^K = 0.1$ and $H_{\pi\pi} = H_{\pi K} = 10$, the ratio in (47) is approximatively 0.89.

The related CP asymmetries will be expressed in terms of the corresponding branching ratios which are scaled by transition form factors as

$$\begin{aligned} Br(\bar{B}^0 \to \pi^+\pi^-) &= C\left(\frac{F_0^{B\to\pi}}{F_0^{B_s\to K}}\right)^2 Br(\bar{B}_s^0 \to K^+\pi^-)\frac{Ph^B_{\pi\pi}}{Ph^{B_s}_{\pi K}}, \\ Br(\bar{B}^0 \to \pi^+K^-) &= C\left(\frac{F_0^{B\to\pi}}{F_0^{B_s\to K}}\right)^2 Br(\bar{B}_s^0 \to K^+K^-)\frac{Ph^{B_d}_{\pi K}}{Ph^{B_s}_{KK}}, \quad (49) \end{aligned}$$

where $C = (m_B^2\tau_{B_s}/m_{B_s}^2\tau_B)$ and $Ph^B_{P_1P_2} = [(1-(m_{P_1}+m_{P_2})^2/m_B^2)(1-(m_{P_1}-m_{P_2})^2/m_B^2]^{1/2}/2m_B$ is the phase space factor. In order to test the SM, one needs to know the form factors which can be obtained from other processes or from theoretical calculations. Alternatively, one can use these relations to obtain the ratio of the form factors using experimental data.

Our calculations in this improved factorization model show that important SU(3) breaking effects arise from the light-cone distributions of mesons in addition to those already present in the decay constants. These effects can only be estimated with large uncertainty because the parameters $\alpha^P_{1,2}$ are not well determined at present. Using the currently allowed ranges we find,

$$A_{CP}(\pi^+\pi^-) \approx -\left(3.1^{+1.9}_{-0.9}\right) A_{CP}(\pi^+K^-)\ , \tag{50}$$

which can also be used to test the SM and the IFM to some extent.

The relations which are independent of $\alpha^i_{1,2}$ parameters and decay constants, such as (48), are more reliable since they do not receive the main SU(3) breaking corrections that we have investigated.

5.2 $B \to PV$ Decays

5.2.1 M_2 is a vector meson

When the vector meson is factored out, as in

$$\bar{B}^0 \to \pi^+\rho^-\ ,\ \bar{B}^0_s \to K^+K^{*-}\ ,\ \bar{B}^0_s \to K^+\rho^-\ ,\ \bar{B}^0 \to \pi^+K^{*-}\ ,$$

the decay amplitudes can be obtained by replacing the r_K factor with $r^*_K = \frac{2m_{K^*}}{m_b}\frac{f^\perp_{K^*}}{f_{K^*}} \approx 0.3$ (and similarly for r_ρ), and by removing the penguin terms $P^{p,EW}_{M_2,3}$ in the expressions for a_6 and a_8 (the vector meson is described only by a twist-2 distribution amplitude). With all these taken into account, we get [6]:

$$\frac{\Delta^{\bar{B}^0}_{\pi^+\rho^-}}{\Delta^{\bar{B}^0_s}_{K^+K^{*-}}} \approx -\frac{m_B}{m_{B_s}}\frac{f^2_\rho}{f^2_{K^*}}\left(\frac{F^{B\to\pi}_1}{F^{B_s\to K}_1}\right)^2 \frac{1-1.25\alpha^\rho_1-0.18\alpha^\rho_2}{1-1.25\alpha^{K^*}_1-0.18\alpha^{K^*}_2}\ . \tag{51}$$

Using the central values of the ranges

$$\alpha^\rho_1 = 0\,,\ \alpha^\rho_2 = 0.16\pm0.09\,,\ \alpha^{K^*}_1 = 0.18\pm0.05\,,\ \alpha^{K^*}_2 = 0.05\pm0.05$$

and taking $f_\rho \approx 0.96 f_{K^*}$ (we extract this ratio $f_\rho/f_K \approx 0.96$ from $\Gamma(\tau^- \to \rho^-\nu)/\Gamma(\tau^- \to K^{*-}\nu)$) we find:

$$\frac{\Delta^{\bar{B}^0}_{\pi^+\rho^-}}{\Delta^{\bar{B}^0_s}_{K^+K^{*-}}} \approx -1.15\left(\frac{F^{B\to\pi}_1}{F^{B_s\to K}_1}\right)^2\ ,\quad \frac{\Delta^{\bar{B}^0_s}_{K^+\rho^-}}{\Delta^{\bar{B}^0}_{\pi^+K^{*-}}} \approx -1.15\left(\frac{F^{B_s\to K}_1}{F^{B\to\pi}_1}\right)^2 . \tag{52}$$

5.2.2 M_1 is a vector meson

When the meson that picks up the spectator is a vector, as for example in

$$\bar{B}^0 \to \rho^+\pi^- \ , \ \bar{B}^0_s \to K^{*+}K^- \ , \ \bar{B}^0 \to \rho^+K^- \ , \ \bar{B}^0_s \to K^{*+}\pi^- \ ,$$

the corresponding decay amplitudes can be obtained by replacing the form factor $F_0^{B\to P}$ with $A_0^{B\to V}$ and r with $-r$. By neglecting the annihilation contributions, the analogue of (52) is [6]

$$\frac{\Delta^{\bar{B}^0}_{\rho^+\pi^-}}{\Delta^{\bar{B}^0_s}_{K^{*+}K^-}} \approx -\frac{m_B}{m_{B_s}}\frac{f_\pi^2}{f_K^2}\left(\frac{A_0^{B\to\rho}}{A_0^{B_s\to K^*}}\right)^2 \frac{1+110\alpha_1^\pi+15.5\alpha_2^\pi}{1+110\alpha_1^K+15.5\alpha_2^K} \ , \qquad (53)$$

and the same for $\Delta^{\bar{B}^0_s}_{K^{*+}\pi^-}/\Delta^{\bar{B}^0}_{\rho^+K^-}$. We observe the large coefficient of α_1 in both the numerator and denominator. Since $\alpha_1^\pi = 0$ and $\alpha_1^K = 0.3 \pm 0.3$, the denominator has a very large uncertainty, making a prediction for this asymmetry impossible within this framework. On the other hand, this provides an opportunity to constrain (or even to determine) α_1^K when the ratio in (53) is measured. However, one may conclude that, in this case, SU(3) breaking is large and estimates are unreliable.

5.3 $B^- \to K^-\eta'$

The relevant decay amplitude for $B^- \to \eta' K^-$ is [4]

$$\begin{aligned} A(B^-\to\eta'K^-) &= -i\frac{G_F}{\sqrt{2}}(m_B^2-m_{\eta'}^2)F_0^{B\to\eta'}(m_K^2)f_K[V_{ub}V_{us}^*a_1(X) \\ &\quad + V_{pb}V_{ps}^*\left(a_4^p(X)+a_{10}^p(X)+r_K(a_6^p(X)+a_8^p(X))\right)] \\ &\quad - i\frac{G_F}{\sqrt{2}}(m_B^2-m_K^2)F_0^{B\to K}(m_{\eta'}^2)f_{\eta'}^u[V_{ub}V_{us}^*a_2(Y) \\ &\quad + V_{pb}V_{ps}^*[(a_3(Y)-a_5(Y))(2+\sigma) \\ &\quad + \left[a_4^p(Y)-\frac{1}{2}a_{10}^p(Y)+r'\left(a_6^p(Y)-\frac{1}{2}a_8^p(Y)\right)\right]\sigma \\ &\quad + \frac{1}{2}\left(a_9(Y)-a_7(Y)\right)(1-\sigma)] \end{aligned} \qquad (54)$$

where $X=\eta' K$ and $Y=K\eta'$, $r' = 2m_{\eta'}^2/(m_b-m_s)(2m_s)$ and $\sigma = f_{\eta'}^s/f_{\eta'}^u$

As it can be noticed, the coefficients a_i are different for the X and Y final states, since they depend on the twist-2 and twist-3 wave functions of the M_2 meson, except for the hard contribution where the wave functions for both M_1 and M_2 are involved.

The twist-2 distribution amplitude $\phi_K(x)$ has the usual expansion in Gegenbauer polynomials, with $\alpha_1^K = 0.3 \pm 0.3$, $\alpha_2^K = 0.1 \pm 0.3$. The corresponding twist-3 amplitude, ϕ_K^p, is 1. In what it concerns the physical states η and η', these are mixtures of SU(3)-singlet and octet components η_0 and η_8 and the corresponding decay constants, in the two-angle mixing formalism, are given by [1]

$$\begin{aligned} f_{\eta'}^u &= \frac{f_8}{\sqrt{6}} \sin\theta_8 + \frac{f_0}{\sqrt{3}} \cos\theta_0 \\ f_{\eta'}^s &= -2\frac{f_8}{\sqrt{6}} \sin\theta_8 + \frac{f_0}{\sqrt{3}} \cos\theta_0 \end{aligned} \tag{55}$$

with $\theta_8 = -22.2^o$, $\theta_0 = -9.1^o$ $f_8 = 168$ MeV and $f_0 = 157$ MeV. These lead to $f_{\eta'}^u = 63.5$ MeV, $f_{\eta'}^s = 141$ MeV while the relevant form factor for the $B \to \eta'$ transition is

$$F_0^{B\to\eta'} = F_0^\pi \left(\frac{\sin\theta_8}{\sqrt{6}} + \frac{\cos\theta_0}{\sqrt{3}} \right) = 0.137 \tag{56}$$

Even the η' flavor singlet meson has a gluonic content [10] which could bring a contribution to the wave function, this is supposed to be small and therefore we employ, in the calculation of $V_{\eta'}$, $P_{\eta',2}^p$ and $P_{\eta',2}^{p,EW}$ in $a_i(Y)$, only the leading twist-2 distribution amplitude

$$\phi_{\eta'} = 6x\bar{x}\,. \tag{57}$$

Also, since the twist-3 quark-antiquark distribution amplitude do not contribute, due to the chirality conservation, the penguin parts in $a_6^p(Y)$ and $a_8^p(Y)$ are missing.

In the improved factorization method, (IFM), we get for the $B^- \to K^-\eta'$ decay the numerical value $Br(B \to K\eta') = 3.65 \cdot 10^{-5}$ which is comparable to other theoretical estimations [1, 4, 10], but is only half of the averaged experimental data. Even the theoretical predictions have shown that the conventional mechanism seams to be dominant, one has to incorporate new contributions in order to increase the $Br(B \to K\eta')$ numerical values.

6 SUSY Gluonic Dipole Contribution

Employing the Minimal Supersymmetric Standard Model (MSSM), one may add to the effective SM Hamiltonian (54), the following SUSY contributions:

$$H_{3-6}^{SUSY} = -i\,\frac{G_F}{\sqrt{2}}(V_{ub}V_{us}^* + V_{cb}V_{cs}^*)\sum_{i=3}^{6} c_i^{SUSY} O_i \tag{58}$$

and

$$H_{7-8}^{SUSY} = -i\,\frac{G_F}{\sqrt{2}}(V_{ub}V_{us}^* + V_{cb}V_{cs}^*)\left(c_{8g}^{SUSY} O_{8g} + c_{7\gamma}^{SUSY} O_{7\gamma}\right), \tag{59}$$

expressed in terms of the usual Standard Model operators O_i and the gluon and photon operators:

$$\begin{aligned} O_{8g} &= \frac{g_s}{8\pi^2}\, m_b \bar{s}\sigma_{\mu\nu}(1+\gamma_5)G^{\mu\nu}b\,, \\ O_{7\gamma} &= \frac{e}{8\pi^2}\, m_b \bar{s}\sigma_{\mu\nu}(1+\gamma_5)F^{\mu\nu}b\,, \end{aligned} \tag{60}$$

The Wilson coefficients are given by [5]

$$\begin{aligned} c_3^{SUSY}(M_{SUSY}) &= -\frac{\alpha_s^2}{2\sqrt{2}\,G_F(V_{ub}V_{us}^* + V_{cb}V_{cs}^*)m_{\tilde{q}}^2}\,\delta_{LL}^{bs} \\ &\times \left(-\frac{1}{9}B_1(x) - \frac{5}{9}B_2(x) - \frac{1}{18}P_1(x) - \frac{1}{2}P_2(x)\right) \\ c_4^{SUSY}(M_{SUSY}) &= -\frac{\alpha_s^2}{2\sqrt{2}\,G_F(V_{ub}V_{us}^* + V_{cb}V_{cs}^*)m_{\tilde{q}}^2}\,\delta_{LL}^{bs} \\ &\times \left(-\frac{7}{3}B_1(x) + \frac{1}{3}B_2(x) + \frac{1}{6}P_1(x) + \frac{3}{2}P_2(x)\right) \\ c_5^{SUSY}(M_{SUSY}) &= -\frac{\alpha_s^2}{2\sqrt{2}\,G_F(V_{ub}V_{us}^* + V_{cb}V_{cs}^*)m_{\tilde{q}}^2}\,\delta_{LL}^{bs} \\ &\times \left(\frac{10}{9}B_1(x) + \frac{1}{18}B_2(x) - \frac{1}{18}P_1(x) - \frac{1}{2}P_2(x)\right) \\ c_6^{SUSY}(M_{SUSY}) &= -\frac{\alpha_s^2}{2\sqrt{2}\,G_F(V_{ub}V_{us}^* + V_{cb}V_{cs}^*)m_{\tilde{q}}^2}\,\delta_{LL}^{bs} \\ &\times \left(-\frac{2}{3}B_1(x) + \frac{7}{6}B_2(x) + \frac{1}{6}P_1(x) + \frac{3}{2}P_2(x)\right), \end{aligned} \tag{61}$$

where the $P_1(x)$, $P_2(x)$, $B_1(x)$, $B_2(x)$ functions, coming from the gluino penguins and box diagrams, can be found in [11] and

$$
\begin{aligned}
c_{8g}^{SUSY}(M_{SUSY}) &= -\frac{\sqrt{2}\pi\alpha_s}{G_F(V_{ub}V_{us}^* + V_{cb}V_{cs}^*)m_{\tilde{g}}^2}\,\delta_{LR}^{bs}\,\frac{m_{\tilde{g}}}{m_b}\,G_0(x)\,, \\
c_{7\gamma}^{SUSY}(M_{SUSY}) &= -\frac{\sqrt{2}\pi\alpha_s}{G_F(V_{ub}V_{us}^* + V_{cb}V_{cs}^*)m_{\tilde{g}}^2}\,\delta_{LR}^{bs}\,\frac{m_{\tilde{g}}}{m_b}\,F_0(x)\,,
\end{aligned}
\qquad (62)
$$

where

$$
\begin{aligned}
G_0(x) &= \frac{x}{3(1-x)^4}\left[22 - 20x - 2x^2 + 16x\ln(x) - x^2\ln(x) + 9\ln(x)\right], \\
F_0(x) &= -\frac{4x}{9(1-x)^4}\left[1 + 4x - 5x^2 + 4x\ln(x) + 2x^2\ln(x)\right]
\end{aligned}
\qquad (63)
$$

In the above relations, $x = m_{\tilde{g}}^2/m_{\tilde{q}}^2$, with $m_{\tilde{g}}$ being the gluino mass and $m_{\tilde{q}}$ an average squark mass, while the factor $\delta^{bs} = \Delta^{bs}/m_{\tilde{q}}^2$, where Δ^{bs} are the off-diagonal terms in the sfermion mass matrices, comes from the expansion of the squark propagator in terms of δ, for $\Delta \ll m_{\tilde{q}}^2$. In principle, the dimensionless quantities δ^{bs}, measuring the size of flavor changing interaction for the $\tilde{s}\tilde{b}$ mixing, are present in all the SUSY corrections to the Wilson coefficients and they are of four types, depending on the L or R helicity of the fermionic partners. A simultaneous analysis, in the full parameter space, for both the LL and LR squark mixings is difficult to be performed. However, when dealing with the SUSY contributions to the Wilson coefficients, one finds major differences between them. In this respect, by computing $\{c_i^{SUSY}\}_{i=\overline{3,6}}$, for $M_{SUSY} = m_{\tilde{q}} = 500$ GeV and $x \approx 1$, we have noticed that these corrections can be neglected. Since the ratios of their values and the SM Wilson coefficients are in the range 10^{-8} to 10^{-6}, they certainly do not bring any significant contribution to the branching ratio. The situation looks different in what it concerns the SUSY Wilson coefficients (62) which are going to play and important role in the next coming discussion. Indeed, by comparing the expression (61) and (62), we notice an enhancement factor of $m_{\tilde{g}}/m_b$ in (62). When $m_{\tilde{g}}$ is of order of few hundred GeV, these SUSY contributions will dominate the SM Wilson coefficients, which are proportional to m_b/m_W^2, and one can anticipate a large effect on the branching ratio, even for small values of δ_{LR}.

By considering only the SUSY corrections (62), we replace the Wilson coefficients c_{8g}^{eff} and $c_{7\gamma}^{eff}$, by the total quantities

$$c_{8g}^{total}[x,\delta] \;=\; c_{8g}^{eff} + c_{8g}^{SUSY}(m_b)\ ,\ \ c_{7\gamma}^{total}[x,\delta] \;=\; c_{7\gamma}^{eff} + c_{7\gamma}^{SUSY}(m_b)\ , \qquad (64)$$

where $c^{SUSY}(m_b)$ have been evolved from $M_{SUSY} = m_{\tilde{g}}$ down to the $\mu = m_b$ scale, using the relations

$$\begin{aligned} c_{8g}^{SUSY}(m_b) &= \eta c_{8g}^{SUSY}(m_{\tilde{g}})\,, \\ c_{7\gamma}^{SUSY}(m_b) &= \eta^2 c_{7\gamma}^{SUSY}(m_{\tilde{g}}) + \frac{8}{3}(\eta - \eta^2) c_{8g}^{SUSY}(m_{\tilde{g}})\,, \end{aligned} \qquad (65)$$

with

$$\eta \;=\; \left(\alpha_s(m_{\tilde{g}})/\alpha_s(m_t)\right)^{2/21} \left(\alpha_s(m_t)/\alpha_s(m_b)\right)^{2/23} \qquad (66)$$

We choose $m_{\tilde{q}} = 500$ GeV, $m_{\tilde{g}} = m_{\tilde{q}}$ and $\delta_{LR}^{bs} \equiv \rho e^{i\varphi}$. Thus, the total branching ratio can be expressed in terms of the parameters ρ and φ as

$$BR^{total} \;=\; 10^{-5}\left(3.65 + 447\rho\cos\varphi + 13670\rho^2 + 13.78\rho\sin\varphi\right)\,, \qquad (67)$$

pointing out, besides the (IFM)-value 3.65×10^{-5}, the SUSY contribution depending on ρ and φ. Now, one is able to allowed values for the SUSY parameters, $\rho \in [0.005, 0.01]$ and $\varphi \in [-3\pi/4, 3\pi/4]$, for accommodating the range within the two extreme experimental data, $BR_{exp}(BaBar) = 7 \times 10^{-5}$ and $BR_{exp}(CLEO) = 8 \times 10^{-5}$. For ρ close to the lowest limit of its interval, the predicted BR^{total}-values lie below the experimental data, while for ρ moving to the central value and $-\pi/4 \le \varphi \le \pi/4$, one gets $BR_{total} \in [7 \times 10^{-5}, 8 \times 10^{-5}]$.

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