

Electroproduction of Soft Pions at Large Momentum Transfers

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based on the collaboration with D. Ivanov and A. Peters

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Outline

- 1 Soft Pion Limit
- 2 Asymptotic Limit $Q^2 \rightarrow \infty$
- 3 Light-Cone Sum Rules
- 4 Structure Functions at $x_B \rightarrow 1$
- 5 Differential Cross Section
- 6 Outlook

Soft Pion Limit

pion electroproduction cross section close to threshold $W \rightarrow W_{\text{th}}$

$$e(l) + p(P) \rightarrow e(l') + \pi^+(k) + n(P'),$$

$$e(l) + p(P) \rightarrow e(l') + \pi^0(k) + p(P').$$

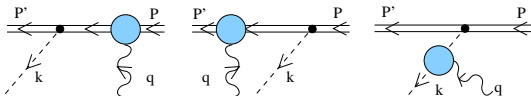
$$W^2 = (P' + k)^2$$

$$W_{\text{th}} = m_N + m_\pi$$

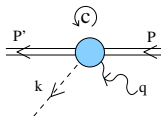
$$Q^2 = -q^2 = -(\ell - \ell')^2$$

can be calculated in the chiral limit $m_\pi \rightarrow 0$, $|\vec{k}| = \mathcal{O}(m_\pi)$, $Q = \mathcal{O}(m_\pi)$ in terms of

- Pion emission from external legs



- Chiral Rotation



$$\langle \pi^a N | j_\mu^{\text{em}} | N \rangle \sim \frac{i}{f_\pi} \langle N | [j_\mu^{\text{em}}, Q_5^a] | N \rangle$$

Kroll, Ruderman '54

Generalized Form Factors

at the threshold

$$\langle \pi N | j_\mu^{\text{em}} | p \rangle = -\frac{i}{f_\pi} \bar{N}(P_2) \gamma_5 \left\{ \left(\gamma_\mu q^2 - q_\mu \not{q} \right) \frac{1}{m_N^2} G_1^{\pi N}(Q^2) - \frac{i \sigma_{\mu\nu} q^\nu}{2m_N} G_2^{\pi N}(Q^2) \right\} N(P_2)$$

related to S-wave multipoles in the PWA

$$E_{0+}^{\pi N} = \frac{\sqrt{4\pi\alpha_{\text{em}}}}{8\pi} \frac{Q^2 \sqrt{Q^2 + 4m_N^2}}{m_N^3 f_\pi} G_1^{\pi N}$$

$$L_{0+}^{\pi N} = \frac{\sqrt{4\pi\alpha_{\text{em}}}}{32\pi} \frac{Q^2 \sqrt{Q^2 + 4m_N^2}}{m_N^3 f_\pi} G_2^{\pi N}$$

e.g. the differential cross section at threshold is given by

$$\left. \frac{d\sigma_{\gamma^*}}{d\Omega_\pi} \right|_{\text{th}} = \frac{2|\vec{k}_f| W}{W^2 - m_N^2} \left[(E_{0+}^{\pi N})^2 + \epsilon \frac{Q^2}{(\omega_\gamma^{\text{th}})^2} (L_{0+}^{\pi N})^2 \right]$$

PCAC + current algebra:

Vainshtein, Zakharov, NPB36(1972)589

Scherer, Koch, NPA534(1991)461

$$\frac{Q^2}{m_N^2} G_1^{\pi^0 p} = \frac{g_A}{2} \frac{Q^2}{(Q^2 + 2m_N^2)} G_M^p, \quad G_2^{\pi^0 p} = \frac{2g_A m_N^2}{(Q^2 + 2m_N^2)} G_E^p,$$

$$\frac{Q^2}{m_N^2} G_1^{\pi^+ n} = \frac{g_A}{\sqrt{2}} \frac{Q^2}{(Q^2 + 2m_N^2)} G_M^n + \frac{1}{\sqrt{2}} G_A, \quad G_2^{\pi^+ n} = \frac{2\sqrt{2} g_A m_N^2}{(Q^2 + 2m_N^2)} G_E^n,$$

- Threshold photoproduction of π^0 is suppressed compared to π^+
- The π^0/π^+ -ratio is rapidly increasing with Q^2

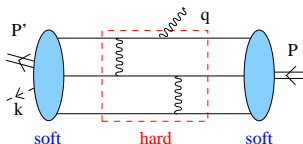
How far in Q^2 can one go?What happens for $Q^2 \gg m_N^2$?

- ◇ Do not address the $\mathcal{O}(m_\pi)$ corrections in this study

very large Q^2

- At large momentum transfers, the pion cannot be soft to the initial and the final state nucleons simultaneously
- The corresponding kinematic condition is $Q^2 > \Lambda_{\text{QCD}}^3/m_\pi$
- Asymptotic limit $Q^2 \rightarrow \infty$ does not commute with the chiral limit $m_\pi \rightarrow 0$

QCD factorization for $Q^2 \gg \Lambda_{\text{QCD}}^3/m_\pi$ **Pobylitsa, Polyakov, Strikman, PRL87(2001)022001:**



- ◇ Probably unrealistic at reachable momentum transfers

Pion-Nucleon Distribution Amplitudes

$$\begin{aligned}
|p \uparrow\rangle &= \frac{\phi_s(x)}{\sqrt{6}} |2u_\uparrow d_\downarrow u_\uparrow - u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle + \frac{\phi_a(x)}{\sqrt{2}} |u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle \\
|p \uparrow \pi^0\rangle &= \frac{\phi_s(x)}{2\sqrt{6}f_\pi} |6u_\uparrow d_\downarrow u_\uparrow + u_\uparrow u_\downarrow d_\uparrow + d_\uparrow u_\downarrow u_\uparrow\rangle - \frac{\phi_a(x)}{2\sqrt{2}f_\pi} |u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle \\
|n \uparrow \pi^+\rangle &= \frac{\phi_s(x)}{\sqrt{12}f_\pi} |2u_\uparrow d_\downarrow u_\uparrow - 3u_\uparrow u_\downarrow d_\uparrow - 3d_\uparrow u_\downarrow u_\uparrow\rangle - \frac{\phi_a(x)}{2f_\pi} |u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle
\end{aligned}$$

Pobylitsa, Polyakov, Strikman; PRL87(2001)022001

Equivalent representation

$$\begin{aligned}
4\langle 0 | \epsilon^{ijk} u_\alpha^i(a_1 z) u_\beta^j(a_2 z) d_\gamma^k(a_3 z) | N(P) \pi \rangle_{\text{twist-3}} &= \\
&= (\gamma_5)_{\gamma\delta} \frac{-i}{f_\pi} \left[V_1^{\pi N} (\not{p} C)_{\alpha\beta} (\gamma_5 N^+)_{\gamma} + A_1^{\pi N} (\not{p} \gamma_5 C)_{\alpha\beta} N_\gamma^+ + T_1^{\pi N} (i\sigma_{\perp p} C)_{\alpha\beta} (\gamma^\perp \gamma_5 N^+)_{\gamma} \right]
\end{aligned}$$

$$\begin{aligned}
V_1^{n\pi^+}(1, 2, 3) &= \frac{1}{\sqrt{2}} \left\{ V_1^n(1, 3, 2) + V_1^n(1, 2, 3) + V_1^n(2, 3, 1) + A_1^n(1, 3, 2) + A_1^n(2, 3, 1) \right\}, \\
A_1^{n\pi^+}(1, 2, 3) &= -\frac{1}{\sqrt{2}} \left\{ V_1^n(3, 2, 1) - V_1^n(1, 3, 2) + A_1^n(2, 1, 3) + A_1^n(2, 3, 1) + A_1^n(3, 1, 2) \right\}, \\
T_1^{n\pi^+}(1, 2, 3) &= \frac{1}{2\sqrt{2}} \left\{ A_1^n(2, 3, 1) + A_1^n(1, 3, 2) - V_1^n(2, 3, 1) - V_1^n(1, 3, 2) \right\}.
\end{aligned}$$

Braun, Ivanov, Lenz, A.Peters; PRD75(2007)014021

Extended to twist-4,5,6

Light-Cone Sum Rules: General Strategy

Balitsky, V.B., Kolesnichenko '86-'88

- consider

$$T_{\nu}^{\pi N}(P, q) = i \int d^4x e^{iqx} \langle 0 | T \{ \eta_p(0) j_{\nu}^{\text{em}}(x) | N(P) \pi^a(k) \rangle$$

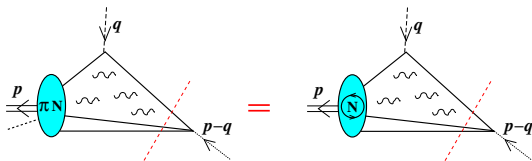
$$\eta_p(x) = \epsilon^{ijk} \left[u^i(x) C \gamma_{\mu} u^j(x) \right] \gamma_5 \gamma^{\mu} d^k(x), \quad \langle 0 | \eta_p | N(P) \rangle = \lambda_p m_N N(P)$$

- take $(P - q)^2 \sim -1 \text{ GeV}^2$ and make a matching between

- (a) The Operator Product Expansion in terms of pion-nucleon DAs

$$\langle 0 | T \{ \eta_p(0) j_{\nu}^{\text{em}}(x) | N(P) \pi^a(k) \rangle = \sum_{\text{twist}} C_{\nu}(x^2, px) \otimes \langle 0 | q(x_1) q(x_2) q(x_3) | N(P) \pi^a(k) \rangle$$

- (b) The dispersion representation in terms of hadronic states



- ◇ Borel transformation to improve convergence
- ◇ Here, pion in initial state (convenient)

Light-Cone Sum Rules: Pion-Nucleon Intermediate States

- New: Semidisconnected pion-nucleon contributions in the intermediate state

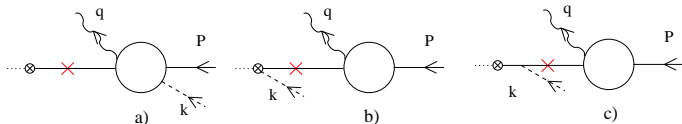


Figure: Schematic structure of the pole terms in the correlation function

- b) and c) correspond to πN coupling to the Ioffe current

$$\langle 0 | \eta_p(0) | N(P' - k) \pi(k) \rangle = \frac{i\lambda_1^p m_N}{2f_\pi} \left[1 - \frac{g_A}{P'^2 - m_N^2} (\not{P}' - \not{k} + m_N) \not{k} \right] \gamma_5 N(P' - k).$$

In the threshold kinematics, with $\delta = m_\pi/m_N$

$$\begin{aligned} T_\nu^{\pi^0 p}(P, q) &= \frac{i\lambda_1^p m_N}{f_\pi} \left\{ \frac{((1 + \delta) \not{P} - \not{q} + m_N) \gamma_5}{m_N^2 - P'^2} \left[(\gamma_\nu q^2 - q_\nu \not{q}) \frac{G_1^{\pi^0 p}}{m_N^2} - \frac{i\sigma_{\nu\mu} q^\mu}{2m_N} G_2^{\pi^0 p} \right] \right. \\ &+ \frac{1}{2} \frac{(1 + \delta) \gamma_5 (\not{P} - \not{q} + m_N)}{[m_N^2(1 + \delta)^2 + \delta Q^2] - P'^2} \left[\gamma_\nu F_1^p - \frac{i\sigma_{\nu\mu} q^\mu}{2m_N} F_2^p \right] \\ &\left. - \frac{(1 + \delta) g_A (\not{P} - \not{q} + m_N) \gamma_5}{[m_N^2(1 + \delta)^2 + \delta Q^2] - P'^2} \left[(\gamma_\nu q^2 - q_\nu \not{q}) G_M^p - \frac{i\sigma_{\nu\mu} q^\mu}{2m_N} 4m_N^2 G_E^p \right] \right\} N(P) \end{aligned}$$

Light-Cone Sum Rules: Pion-Nucleon Intermediate States — *cont.*

- The semidisconnected πN contributions can be included in the continuum if

$$m_\pi Q^2 > m_N(s_0 - m_N^2) \quad \Rightarrow \quad Q^2 > 7 \text{ GeV}^2 \quad [\sim \Lambda_{\text{QCD}}^3/m_\pi]$$

- Otherwise they have to be taken into account explicitly

$$\begin{aligned} \frac{Q^2}{m_N^2} G_1^{\pi^0 p} &= \frac{e^{m_N^2/M^2}}{2\lambda_1^p} \mathbb{B}_{p/2} [\mathcal{A}^{\pi^0 p}](M^2, Q^2) - \frac{1}{2} e^{-\delta(2m_N^2+Q^2)/M^2} \left[F_1^p(Q^2) - \frac{g_A Q^2}{Q^2 + 2m_N^2} G_M^p(Q^2) \right] \\ G_2^{\pi^0 p} &= -\frac{e^{m_N^2/M^2}}{\lambda_1^p} \mathbb{B}_{p/2} [\mathcal{B}^{\pi^0 p}](M^2, Q^2) + e^{-\delta(2m_N^2+Q^2)/M^2} \left[\frac{1}{2} F_2^p(Q^2) + \frac{2g_A m_N^2}{Q^2 + 2m_N^2} G_E^p(Q^2) \right] \\ \frac{Q^2}{m_N^2} G_1^{\pi^+ n} &= \frac{e^{m_N^2/M^2}}{2\lambda_1^p} \mathbb{B}_{p/2} [\mathcal{A}^{\pi^+ n}](M^2, Q^2) - \frac{1}{\sqrt{2}} e^{-\delta(2m_N^2+Q^2)/M^2} \left[F_1^n(Q^2) - \frac{g_A Q^2}{Q^2 + 2m_N^2} G_M^n(Q^2) \right] \\ G_2^{\pi^+ n} &= -\frac{e^{m_N^2/M^2}}{\lambda_1^p} \mathbb{B}_{p/2} [\mathcal{B}^{\pi^+ n}](M^2, Q^2) + e^{-\delta(2m_N^2+Q^2)/M^2} \left[\frac{1}{\sqrt{2}} F_2^n(Q^2) + \frac{2\sqrt{2}g_A m_N^2}{Q^2 + 2m_N^2} G_E^n(Q^2) \right] \end{aligned}$$

where $\mathcal{A}(P'^2, Q^2)$ and $\mathcal{B}(P'^2, Q^2)$ are the invariant functions defined as

$$z^\nu \Lambda^+ T_\nu^{\pi N}(P, q) = \frac{i}{f_\pi} (pz + kz) \gamma_5 \left\{ m_N \mathcal{A}(P'^2, Q^2) + \not{q}_\perp \mathcal{B}(P'^2, Q^2) \right\} N^+(P)$$

Light-Cone Sum Rules: Results

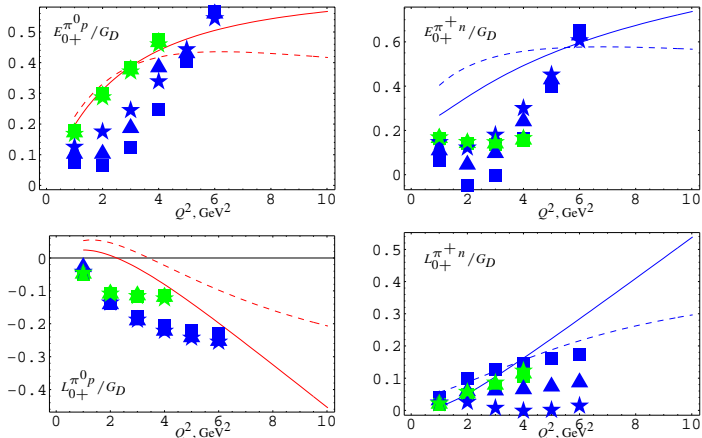


Figure: The LCSR prediction for E_{0+} and L_{0+} at threshold, compared to the extrapolations of the existing PWA (private communication from I. Strakovsky)

◇ normalized to the dipole formula $G_D = 1/(1 + Q^2/0.71)^2$

Away from the threshold . . .

for $W - W_{\text{th}} \ll m_\pi$ accept

$$\begin{aligned}
 \langle N(P') \pi(k) | j_\mu^{em}(0) | P(P) \rangle = & \\
 = & -\frac{i}{f_\pi} \bar{N}(P') \gamma_5 \left\{ \left(\gamma_\mu q^2 - q_\mu \not{q} \right) \frac{1}{m_N^2} G_1^{\pi N}(Q^2) - \frac{i \sigma_{\mu\nu} q^\nu}{2m_N} G_2^{\pi N}(Q^2) \right\} N(P) \\
 & + \frac{ic_\pi g_A}{2f_\pi [(P'+k)^2] - m_N^2} \bar{N}(P') \not{k} \gamma_5 (\not{P}' + m_N) \left\{ F_1^p(Q^2) \left(\gamma_\mu - \frac{q_\mu \not{q}}{q^2} \right) + \frac{i \sigma_{\mu\nu} q^\nu}{2m_N} F_2^p(Q^2) \right\} N(P)
 \end{aligned}$$

- ◇ **S-wave: generalized form factors from LCSR**
- ◇ **P-wave: pion emission from the final state nucleon; exact in chiral limit**
- ◇ **Eventually can take into account the final state interactions**

$$G_1^{\pi N}(Q^2) \rightarrow G_1^{\pi N}(Q^2, W) \equiv G_1^{\pi N}(Q^2) [1 + i t_{\pi N}]$$

Structure Functions at $x_B \rightarrow 1$

$$F_1(W, Q^2) = \frac{\beta(W)}{(4\pi f_\pi)^2} \sum_{\pi^0, \pi^+} \left\{ \frac{Q^2 + 4m_N^2}{2m_N^4} |Q^2 G_1^{\pi N}|^2 + \frac{c_\pi^2 g_A^2 W^2 \beta^2(W)}{8(W^2 - m_N^2)^2} Q^2 m_N^2 G_M^2 \right\}$$

$$F_2(W, Q^2) = \frac{\beta(W)}{(4\pi f_\pi)^2} \sum_{\pi^0, \pi^+} \left\{ \frac{Q^2}{m_N^4} \left(|Q^2 G_1^{\pi N}|^2 + \frac{m_N^2}{4} Q^2 |G_2^{\pi N}|^2 \right) + \frac{c_\pi^2 g_A^2 W^2 \beta^2(W) Q^2 m_N^2}{4(W^2 - m_N^2)^2} \left(\frac{Q^2 G_M^2 + 4m_N^2 G_E^2}{Q^2 + 4m_N^2} \right) \right\}$$

$$g_1(W, Q^2) = \frac{\beta(W)}{(4\pi f_\pi)^2} \sum_{\pi^0, \pi^+} \left\{ \frac{Q^2}{2m_N^4} \left[|Q^2 G_1^{\pi N}|^2 - m_N^2 \text{Re}(Q^2 G_1^{\pi N} G_2^{*, \pi N}) \right] + \frac{c_\pi^2 g_A^2 W^2 \beta^2(W)}{8(W^2 - m_N^2)^2} Q^2 m_N^2 G_M F_1^p \right\}$$

$$g_2(W, Q^2) = -\frac{\beta(W)}{(4\pi f_\pi)^2} \sum_{\pi^0, \pi^+} \left\{ \frac{Q^2}{2m_N^4} \left[|Q^2 G_1^{\pi N}|^2 + \frac{1}{4} Q^2 \text{Re}(Q^2 G_1^{\pi N} G_2^{*, \pi N}) \right] + \frac{c_\pi^2 g_A^2 W^2 \beta^2(W)}{32(W^2 - m_N^2)^2} Q^4 G_M F_2^p \right\}$$

$$\beta(W) = \frac{|\vec{k}_f|}{W},$$

$$x_B = \frac{Q^2}{Q^2 + W^2 - m_N^2}$$

SLAC E136

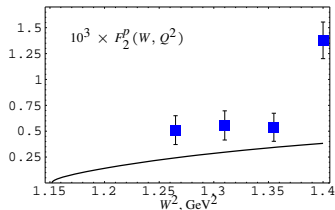
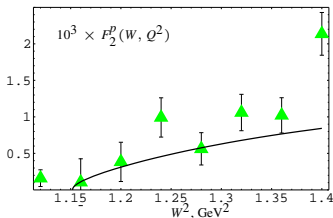
P. E. Bosted *et al.*; PRD49(1994)3091

Figure: The structure function $F_2^p(W, Q^2)$ as a function of W^2 scaled by a factor 10^3 compared to the SLAC E136 data at the average value $Q^2 = 7.14 \text{ GeV}^2$ (left panel) and $Q^2 = 9.43 \text{ GeV}^2$ (right panel).

Differential Cross Section

For unpolarized protons, the virtual photon cross section is

$$d\sigma_{\gamma^*} = \frac{\alpha_{\text{em}}}{8\pi} \frac{k_f}{W} \frac{d\Omega_\pi}{W^2 - m_N^2} |\mathcal{M}_{\gamma^*}|^2$$

with

$$|\mathcal{M}_{\gamma^*}|^2 = M_T + \epsilon M_L + \sqrt{2\epsilon(1+\epsilon)} M_{LT} \cos(\phi_\pi) + \epsilon M_{TT} \cos(2\phi_\pi) + \lambda \sqrt{2\epsilon(1-\epsilon)} M'_{LT} \sin(\phi_\pi)$$

$$f_\pi^2 M_T = \frac{4\bar{k}_i^2 Q^2}{m_N^2} |G_1^{\pi N}|^2 + \frac{c_\pi^2 g_A^2 \bar{k}_f^2}{(W^2 - m_N^2)^2} Q^2 m_N^2 G_M^2 + \cos\theta \frac{c_\pi g_A |k_i| |k_f|}{W^2 - m_N^2} 4Q^2 G_M \text{Re } G_1^{\pi N}$$

$$f_\pi^2 M_L = \bar{k}_i^2 |G_2^{\pi N}|^2 + \frac{4c_\pi^2 g_A^2 \bar{k}_f^2}{(W^2 - m_N^2)^2} m_N^4 G_E^2 - \cos\theta \frac{c_\pi g_A |k_i| |k_f|}{W^2 - m_N^2} 4m_N^2 G_E \text{Re } G_2^{\pi N}$$

$$f_\pi^2 M_{LT} = -\sin\theta \frac{c_\pi g_A |k_i| |k_f|}{W^2 - m_N^2} Q m_N [G_M \text{Re } G_2^{\pi N} + 4G_E \text{Re } G_1^{\pi N}]$$

$$f_\pi^2 M_{TT} = 0,$$

$$f_\pi^2 M'_{LT} = -\sin\theta \frac{c_\pi g_A |k_i| |k_f|}{W^2 - m_N^2} Q m_N [G_M \text{Im } G_2^{\pi N} - 4G_E \text{Im } G_1^{\pi N}]$$

- ◇ $M_{TT} = 0$: no D-wave; tests quality of the approximation
- ◇ M'_{LT} : single-spin asymmetry; arises because of FSI, calculable

Miscellaneous Results

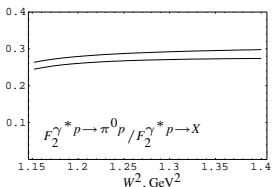


Figure: The fraction of $\pi^0 p$ in $F_2^p(W, Q^2)$ for $Q^2 = 3 \text{ GeV}^2$ (upper curve) and $Q^2 = 9 \text{ GeV}^2$ (lower curve)

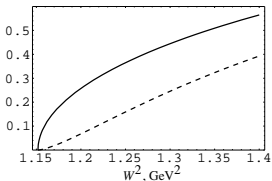


Figure: S-wave (solid) vs. P-wave (dashed) for $F_2^p(W, Q^2)$ at $Q^2 = 7.14 \text{ GeV}^2$

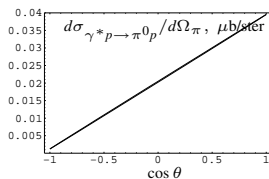


Figure: Differential cross section $d\sigma_{\gamma^* p \to \pi^0 p} / d\Omega_\pi$ for $\phi_\pi = 135 \text{ grad}$, $Q^2 = 4.2 \text{ GeV}^2$ and $W = 1.11 \text{ GeV}$

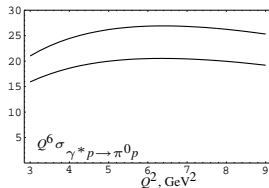


Figure: Integrated cross section $Q^6 \sigma_{\gamma^* p \to \pi^0 p}$ for $W = 1.11 \text{ GeV}$ (lower curve) and $W = 1.15 \text{ GeV}$ (upper curve)

Outlook

physics issues:

- a novel object: generalized form factor; a complete direct measurement is possible
- interesting interplay between perturbative QCD and chiral symmetry breaking
- new connection between QCD and traditional hadron physics (PWA)
- an (almost) untouched terrain...

theory:

- detailed predictions possible near to threshold
- present LCSR accuracy probably 50%; can be improved to 20%(?)
- elastic πN rescattering can be taken into account
- can be incorporated in full PWA

experiment:

- Have to go below $W = 1150 - 1180$ MeV
- good energy resolution crucial
- 4π geometry not necessary
- proton polarization not necessary but would help a lot

Perfectly suited for the JLab 12 GeV upgrade physics program